AD-A064 330 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH--ETC F/G 17/4

IMPROVED ANTI-JAM PERFORMANCE IN AN ADAPTIVE SPREAD SPECTRUM CO--ETC(U)

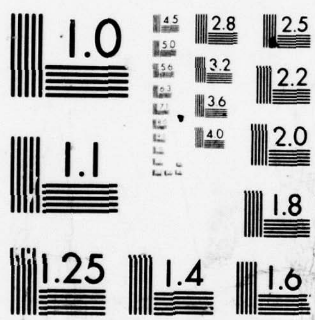
DEC 78 E R SIVE

UNCLASSIFIED AFIT/GE/EE/78D-40 NL

1 OF 1

AD A064330

END
DATE
FILMED
4--79
DDC



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

14 AFIT/GE/EE/78D-40

1

LEVEL II

6 IMPROVED ANTI-JAM PERFORMANCE
IN AN ADAPTIVE SPREAD SPECTRUM
COMMUNICATION SYSTEM,

9 Master's THESIS,

AFIT/GE/EE/78D-40

10 Emanuel R. Sive
Captain USAF

11 Dec 78 12 79p.

16 2277

012 225

120

AFIT/GE/EE/78D-40

IMPROVED ANTI-JAM PERFORMANCE IN AN ADAPTIVE SPREAD SPECTRUM COMMUNICATION SYSTEM

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Emanuel R. Sive
Captain USAF
Graduate Electrical Engineering

December 1978

Approved for public release; distribution unlimited

Preface

An adaptive receiver and a spread spectrum receiver are combined into one improved jam resistant communication's system. This need for increased jam resistance is attributed to the reliance on command and control communications by our national and military leaders in times of hostilities. This thesis investigates the performance of an adaptive spread spectrum communication's system in an attempt to insure jam resistant communications when needed.

Thanks are due to Major Joe Carl and Captain Hadley for their assistance and technical advice. My wife, Joann, deserves acknowledgement for her patience, encouragement, and assistance in the preparation of this report. Also, Margaret Voigt deserves recognition for her typing. A special thanks goes to Captain Stan Robinson for his patience, guidance, and understanding throughout the entire past year.

Emanuel R. Sive

Contents

		Page
Preface		ii
List of Figures		iv
List of Tables		vi
Abstract		vii
I.	Introduction	1
II.	Autocorrelation Peak Degradation	10
	Spread Spectrum Signal Model	10
	Filter Centered on Spread Spectrum Signal	13
	Filter Offset by Fifty Percent from Center of Spread Spectrum Signal	23
II.	Signal-to-Noise Ratio	30
	Spread Spectrum Receiver Without a Colored Noise Filter	30
	Spread Spectrum Receiver with a Colored Noise Filter	36
IV.	Intersymbol Interference	41
V.	Conclusions	45
Bibliography		49
Appendix A		50
Appendix B		52
Appendix C		54
Appendix D		56
Appendix E		57
Appendix F		60

List of Figures

Figure		Page
1	Adaptive Receiver	4
2	Spread Spectrum Receiver	4
3	Typical c(t) Waveform	11
4	Autocorrelation Function of c(t) . . .	11
5	Block Diagram of Colored-Noise Filter and Matched Filter	11
6	Plot of $H_{pm}(f)$ Centered on f_m	16
7	Baseband Model of Colored-Noise Filter and Matched Filter; (a) Time Domain, and (b) Frequency Domain	16
8	Normalized Plot of (a) S(f) and (b) Inverse Fourier Transform of S(f)	20
9	Baseband Representation of (a) Colored-Noise Filter, $H_{sc}(f)$; and, (b) Inverse Fourier Transform of $H_{sc}(f)$	20
10	Plot of Peak Degradation versus Percent Percent B/W (Filter Centered)	21
11	Spread Spectrum Autocorrelation Function (a) Without Colored-Noise Filter, and (b) With Colored-Noise Filter	22
12	Colored-Noise Filter Offset from f_m . .	24
13	Plot of (a) Spread Spectrum Power Density Function and (b) Narrowband Colored-Noise Filter Centered on Δf	24
14	Plot of Peak Degradation versus Percent B/W (Filter Offset) (a) Upper Bound, (b) Average, and (c) Lower Bound	28
15	Correlation Receiver	32
16	Matched Filter Receiver	32

Figure		Page
17	Receiver with Colored-Noise Filter; (a) Correlator Representation; and (b) Matched Filter Representation	37
18	Receiver Block Diagram with Color-Noise Filter for Noise Power Calculations . . .	37
19	Plot of Eq (64) Normalized by t_sB	44
F1	Block Diagram of Spread Spectrum Receiver Used for Noise Power Calculations	61
F2	Plot of Wideband Jammer's Power Spectrum Going through Receiver of Figure F1; (a) $J_1(f)$, (b) $J_2(f)$, and (c) $J_o(f)$	68

List of Tables

Table		Page
I	Effect of Colored-Noise Filter Centered on Spread Spectrum Signal	51
II	Results of Using Colored-Noise Filter Offset by 50 Percent	53
III	Upper Bound on d (50 Percent Offset) . .	55
IV	Lower Bound on d (50 Percent Offset) . .	56

Abstract

This thesis investigates the anti-jam performance of an adaptive spread spectrum communication's system. The adaptive filter is assumed to be ideal in that it perfectly locates the jamming signal and adapts it's bandwidth to exactly filter out the jamming signal. The colored-noise (jammer) and bandlimited white noise are considered to have flat power spectral densities within their respective bandwidths. The spread spectrum technique considered is that of direct sequence modulation. Three criteria are used to evaluate the system: autocorrelation peak degradation, signal-to-noise ratio, and intersymbol interference. The results indicate that an adaptive spread spectrum communication's system is able to provide an increased jam resistant communication system.

IMPROVED ANTI-JAM PERFORMANCE IN AN ADAPTIVE SPREAD SPECTRUM COMMUNICATION SYSTEM

I. Introduction

Military communications provide our national and military leaders the means of controlling the military might of our nation. In time of national conflict, it becomes vital for our military communication systems to perform properly. It can be expected that our nation's enemies will make every effort to either impede (jam) or destroy our communication systems. Therefore, in order to achieve an increased jam resistant capability in a hostile (jamming) environment, it is necessary to improve our current communication systems.

One method to achieve improved communications reliability is the use of adaptive receiver communication techniques (Ref 5). The adaptive receiver techniques makes use of a linear receiver which has a bandpass filter, a sampler, and a tapped delay line (TDL) filter. Any signal which passes through this receiver is first filtered to remove any signals which are outside the desired information signal's narrow frequency band. The output of the filter is samples and sent to the TDL filter. Each sample which is

stored at one of the TDL filter taps is multiplied by that particular tap's gain value and a sum of all the tap product terms is made. This sum is an estimate of the desired information signal. This estimated signal is compared to an estimated desired signal and an error signal is produced. This error signal, which corresponds to the noise added to the desired signal, is used as an input to the tap gain adjustment circuitry, which can effectively adjust the filter's characteristics in order to filter out the unwanted signals from the actual received signal. The output of the system, ideally, is the desired information signal without the interference which had been added during the transmission process. The adaptive receiver is sometimes called a "smart" receiver since it can automatically determine what the unwanted signal is and reduce it in strength without affecting the desired information signal. This technique has been limited in the past because of technology and the difficulty of obtaining a close estimate of the desired signal. Electronic devices such as the TDL filter were unable to respond to a signal fast enough to adequately filter out the undesired signals. Technological advances in the area of an electronic device called a charge coupled device (CCD) have made it possible to perform this kind of sophisticated signal processing with the speed necessary for adaptive processing (Ref 5:484).

A second method of achieving improved anti-jam communication uses spread spectrum communication techniques

(Ref 1, 3). The inherent characteristics of a spread spectrum communications system provide a means of protecting communications from detection, demodulation, and interference by an unauthorized receiver and/or jammer. A spread spectrum system achieves these characteristics by spreading the desired information signal to be transmitted over a wide band of frequencies. This wide band of frequencies is obtained by using one of the four basic-spreading modulation methods: direct sequence, frequency hopping, pulse-FM (chirp), or time hopping (Ref 3:1). This paper will consider only direct sequence modulation.

A direct sequence modulated spread spectrum communications system is one of the four possible pseudonoise modulation methods (Ref 1:13). As the name implies, the transmitted signal is generated by modulating an information bearing signal with a high speed code sequence. The code sequence is a binary sequence with a length long enough and bit (code chip) duration time short enough to thinly spread the relatively narrow-band information signal over a band of frequencies typically 10 to 100 times the original bandwidth. The modulation is accomplished by first digitizing the information signal, multiplying it to the much higher bit rate code sequence, and using the resulting signal to modualte a radio frequency carrier.

The spread spectrum receiver accomplishes the frequency despreading by correlating the received spread spectrum signal with a similar, locally generated, reference

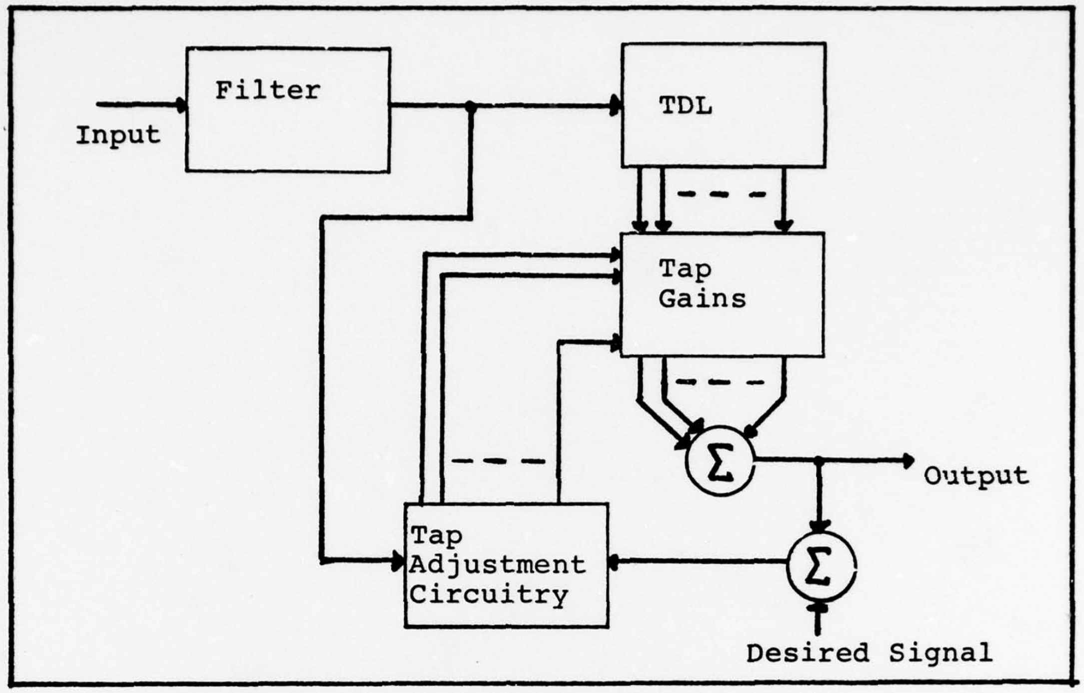


Figure 1. Adaptive receiver.

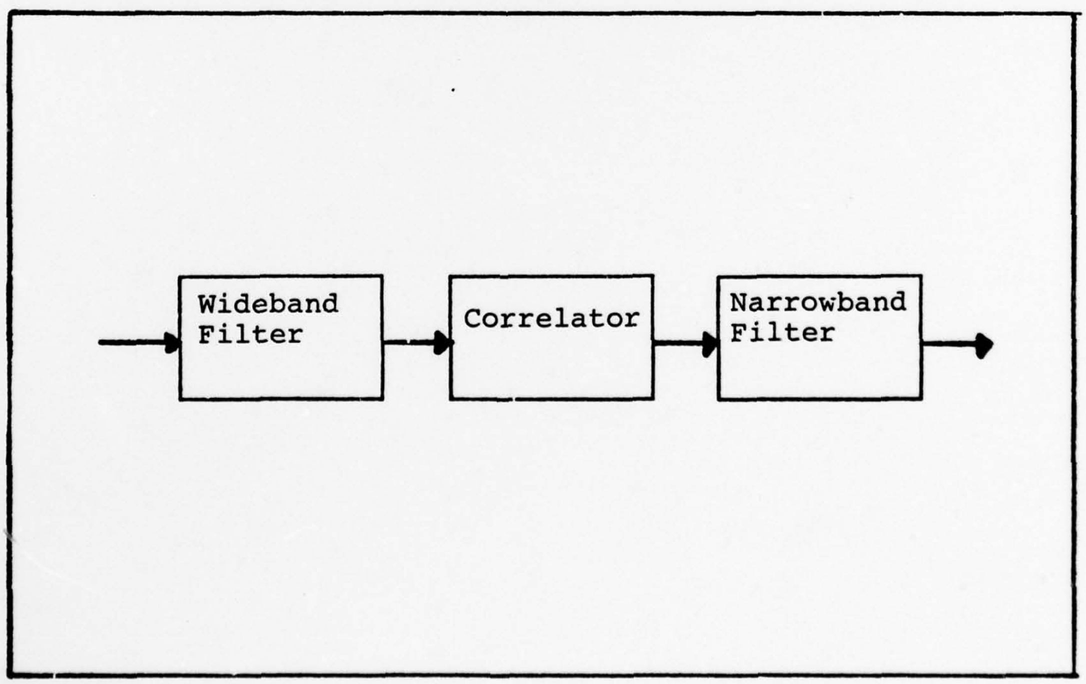


Figure 2. Spread spectrum receiver

signal. The correlation of the two matched signals causes the frequency spread information signal to be compressed back to its original bandwidth (before spreading). The de-spread signal is then passed through a lowpass filter. Signal processing from this point continues as if the spread spectrum technique had not been used. Any undesired signal which is received by the spread spectrum receiver that is not matched to the locally generated reference signal will be frequency spread as was the original information bearing signal at the transmitter. The amount of spreading of the undesired signal is equal to the bandwidth of the locally generated reference signal.

The amount of process gain, or signal-to-noise improvement factor, achieved by a spread spectrum system is given as the ratio of the bandwidth of the transmitted spread spectrum signal divided by the information (data) rate (Ref 1:23). However, in order to calculate a jamming margin, two additional performance measures must be included. These are internal receiver processing loss and required receiver output level. Typically, these values are 2db and 10db respectively. If we limit the processing gain to 27db because of the required data rate and maximum transmission bandwidth restrictions, we have a jamming margin of 15db (process gain minus processing loss minus requied receiver operation level)(Ref 1:8; 2:16,20). Use of a practical error correcting code has the effect of adding 3db to the jamming margin, since the code is able

to correct some of the receiver's errors. Therefore, a jamming margin of 18 dB is achieved with a typical spread spectrum communication system used in conjunction with error correcting codes.

Although this method of providing anti-jam communications has been effective, it has its limitations. It can be overpowered by strong jamming signals. It is also reasonable to expect that hostile forces will be building more efficient and more powerful jamming equipment in the future. Thus, jamming to desired signal levels of up to 60 dB are design goals in a practical military spread spectrum communications system. To achieve this margin of protection, an additional 42 dB (60 dB-18 dB) of protection is required. This additional protection will have to be provided by other means, for example, the use of antenna directivity and antenna nulls, electronic interference cancellation, and/or colored noise filters.

Antenna cancellation techniques make use of the directivity of an antenna and the ability to place, or steer, a null in the direction of the interferring signal. Such techniques are limited because of geographic considerations. A jammer located in-line between the desired transmitter and desired receiver would prohibit the use of a null and neutralize the advantage of a directional antenna. However, the probability of an interference source operating for any significant amount of time in-line with the desired transmitter is small. Therefore, a 20 dB to 30 dB additional

jamming margin can be expected by the use of antenna cancellation techniques (Ref 2:22-24).

Effective electronic interference cancellation techniques require the generation of a duplicate interference signal. This duplicate is subtracted from the received signal leaving only the desired signal. The problem encountered is that the interferring signal to be duplicated must be relatively unsophisticated and there must be only a few of them. This method requires the use of a signal generator and subtractor for each interference signal or a highly directional antenna, a signal processor, and a subtractor. The latter is used when the interferring signal is received by a second receiver antenna and used to form the duplicate interference signal. The complexity, cost, and limitations of this technique in the past have made this method impractical to use (Ref 1:9). However, current advances in the technology of electronic devices may allow this type of processing to become practical.

The last possibility is the use of colored-noise filters. These filters are limited to rejecting interference signals which occupy a narrow band of frequencies compared to the spread spectrum signal's frequency band. The use of wideband filters would not only reject the interferring signal but also a large portion of the desired spread spectrum signal. The narrowband colored-noise filter is positioned in the receiver portion of the system prior to the correlator (or matched filter equivalent).

With one exception the most effective jammer against a spread spectrum system is a narrowband jammer (Ref 1:142). The only exception is an interferring signal that is correlated or matched to the spread spectrum code. The reason that a continuous wave jammer is so effective is that the bandwidth of the interference signal after the correlator is equal to its own bandwidth plus the bandwidth of the locally generated reference signal. Thus a wideband interference signal will have its power spread over a wider bandwidth than a narrowband interference signal. When the frequency spread interference signal is passed through the lowpass filter following the correlator, a portion of its power will be filtered out. Therefore, the larger the interference signal's bandwidth, the smaller the amount of power that falls within the passband of the filter.

In an effort to find what method or combination of methods is best suited to the suppression of interference, an investigation of spread spectrum techniques in an idealized adaptive system will be the subject of this paper. The use of a direct sequence spread spectrum code operating on binary data solves the problem of obtaining a duplicate of the desired signal required by the adaptive receiver. The spread spectrum code or its complement becomes the desired signal. Since the greatest threat to a spread spectrum system is a narrowband jammer, an investigation of the use of a programable tapped delay line

as an adaptive narrowband filter in a spread spectrum receiver is made. The adaptive narrowband filter is considered ideal in that it can perfectly locate the jamming signal and adjust to its bandwidth. The adaptive circuit of Figure 1 can perform the function of the adaptive narrowband filter (colored-noise filter) and is located ahead of the spread spectrum receiver's correlator.

Three criteria will be used to evaluate the performance of this system: the amount of desired signal autocorrelation peak degradation introduced (Ref 3); the signal-to-noise ratio improvement factor (Ref 4); and amount of degradation caused by intersymbol interference. A spread spectrum signal model is first developed in order to use the criteria.

II. Autocorrelation Peak Degradation

Spread Spectrum Signal Model

A spread spectrum signal, $m(t)$, at radio frequency can be represented over a signalling interval by

$$m(t) = i(t)c(t)\cos(w_m t) \tag{1}$$

where,

$i(t)$	is the information bit (±1)
$c(t)$	is the spread spectrum code at baseband
w_m	is equal to $2\pi f_m$, where f_m is the modulation frequency

A plot of a typical spread spectrum code, $c(t)$ is shown in Figure 3.

The energy in $m(t)$ is calculated as follows

$$\begin{aligned} E &= \int_0^T [i(t)c(t)\cos(w_m t)]^2 dt \\ &= \int_0^{Nts} |c(t)|^2 \cos^2(w_m t)dt \\ &= \int_0^{Nts} A^2(1/2 + (1/2)\cos(2w_m t))dt \\ &= \int_0^{Nts} A^2/2\, dt + \int_0^{Nts} (1/2)A^2\cos(2w_m t)dt \\ &= \frac{A^2Nt}{2} \end{aligned} \tag{2}$$

where,

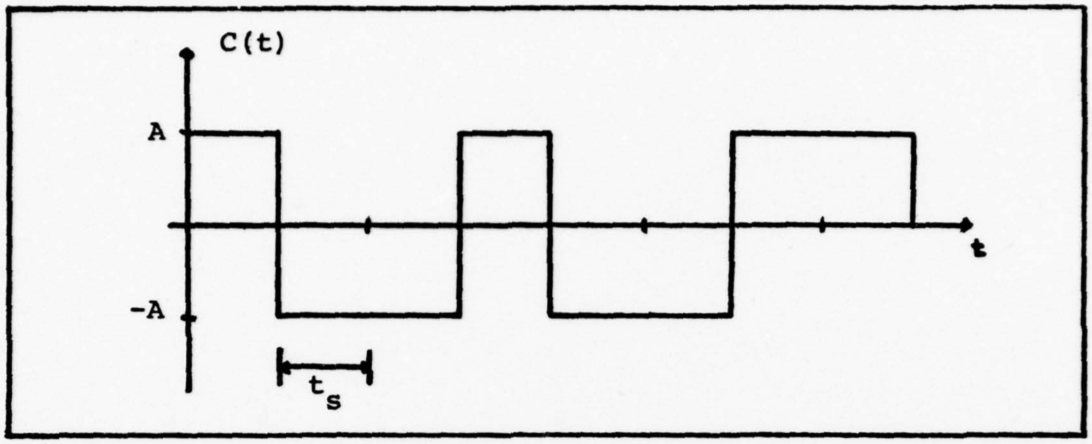


Figure 3. Typical c(t) waveform..

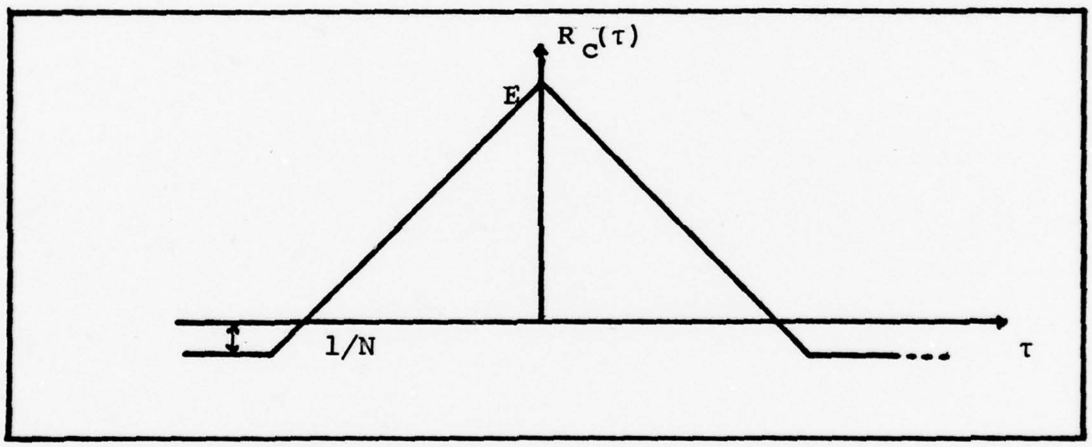


Figure 4. Autocorrelation function of c(t).

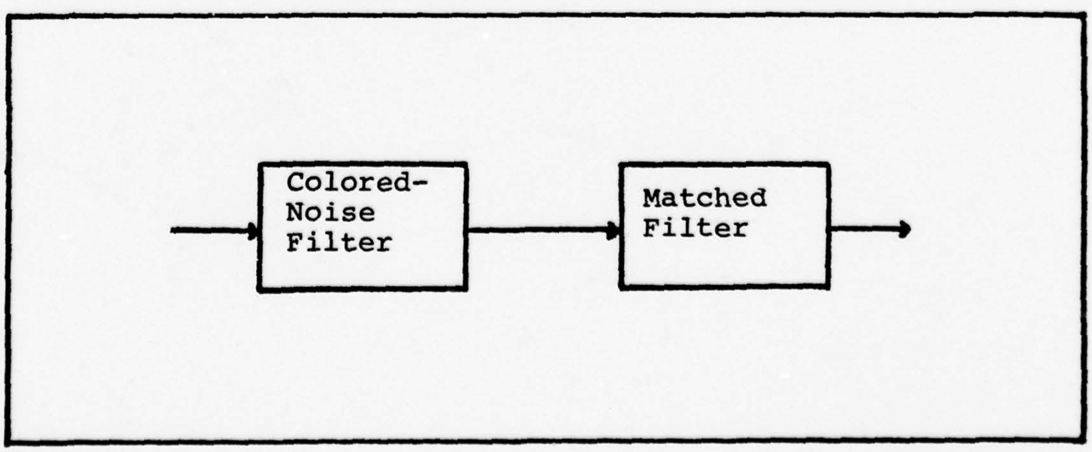


Figure 5. Block diagram of colored-noise filter and matched filter.

t_s is the chip width of the code

N is the number of chips in the code

A is the magnitude of each chip

T is much greater than $(f_m)^{-1}$

One property of the time autocorrelation function, $R(\tau)$, is that its magnitude at τ equal to zero is the total energy contained in the signal. Since the code is a periodic sequence of pulses it will have a triangular shape with a base length equal to twice the chip width of the code. This occurs after many autocorrelations have taken place. Another property of the autocorrelation function is that it is an even function. From this information it is possible to sketch $R_c(\tau)$ in Figure 4 (Ref 7:18, 47, 229). The triangular pulse is repeated every Nt_s seconds.

The Fourier transform of the autocorelation function is the energy density spectrum which is equal to

$$
\begin{aligned}
S(f) &= F[A^2Nt_s/2)(1-\frac{|t|}{t_s}) \text{ rect } (t/2t_s)] \\
&= E \ F[(1-(|t|/t_s)) \text{ rect } (t/2t_s)] \\
&= E \ t_s \ \text{sinc}^2(t_sf) \qquad (3)
\end{aligned}
$$

where,

rect(•) is a rectangle function

E is defined by Eq (2)

Eqs (1), (2), and (3) as well as Eq (10) of Appendix E become the models to be used in the analysis that follows. Also, the receiver is considered to be coherent and

synchronized to the codeword. Therefore, the received signal is either the modulated codeword or the modulated codeword complement. The received signal is considered to be completely determined.

<u>Filter Centered on Spread Spectrum Signal</u>

Armed with the information from the previous section we will proceed to calculate the amount the peak of the autocorrelation function is reduced (degraded) by the use of a colored-noise filter in the spread spectrum system. Figure 5 shows a block diagram of the filter and the matched filter used in the receiver section of the spread spectrum system.

The colored-noise filter is an ideal bandstop filter with a bandwidth, B, that is much smaller than the bandwidth of the spread spectrum signal. The matched filter is matched to the spread spectrum code.

The ideal band stop filter has a transfer function equal to $H_{sm}(f)$, where

$$H_{sm}(f) = \begin{matrix} 0 & f_m - B/2 \le |t| \le f_m + B/2 \\ 1 & \text{else} \end{matrix} \tag{4}$$

Using an ideal bandpass filter, with transfer function $H_{pm}(f)$, instead of the ideal bandstop filter yields the following equality

$$H_{sm}(f) = 1 - H_{pm}(f) \tag{5}$$

where,

$$H_{pm}(f) = \begin{cases} 1 & f_m - B/2 \leq |f| \leq f_m + B/2 \\ 0 & \text{else} \end{cases} \qquad (6)$$

The spread spectrum signal is centered on a frequency f_m. In order to find the baseband representation of the ideal narrowband filter, consider Figure 6. The time domain representation of the colored-noise filter is the inverse Fourier transform of its frequency domain representation. Therefore,

$$h_{pm}(t) = F^{-1}[\mathrm{rest}(\tfrac{f+f_m}{B}) + \mathrm{rect}\ \tfrac{f-f_m}{B}]$$

$$= F^{-1}[\mathrm{rect}(\tfrac{f}{B})][e^{+jw_m t} + e^{-jw_m t}]$$

$$= B\ \mathrm{sinc}\ (Bt)\ 2\cos(w_m t)$$

$$= \mathrm{Re}\ [2B\ \mathrm{sinc}\ (Bt)\ e^{jw_m t} \qquad (7)$$

Defining $h_{pc}(t)$ as

$$h_{pc}(t) = 2\ B\ \mathrm{sinc}\ (Bt) \qquad (8)$$

Where $h_{pc}(t)$ is the baseband representation for the ideal bandpass filter centered on f_m. Taking the inverse Fourier transform of Eq (8) yields

$$H_{pc}(f) = 2\ \mathrm{rect}(\tfrac{f}{B}) \qquad (9)$$

The baseband representation of the colored-noise filter now becomes $H_{sc}(f)$

$$H_{sc}(f) = K - H_{pc}(f)$$

$$= 2-2 \text{ rect } (f/B) \qquad (10)$$

where K is the constant obtained from moving $H_{sm}(f)$ to baseband.

It is now possible to construct the baseband model, Figure 7, of the colored-noise filter and the matched filter.

Using:

$$\left.\begin{array}{l} c(t) \leftrightarrow C(f) \\ R(\tau) \leftrightarrow S(f) \end{array}\right\} \text{ Fourier transform pairs} \qquad (11)$$

$C^*(f)$ is the complex conjugate of $C(f)$

$R(\tau)$ is the autocorrelation function of $c(t)$

$S(f)$ is the Fourier transform of $R(\tau)$

From Figure 7:

$$C''(f) = C(f)\, H_{sc}(f)$$

$$= C(f)\,(2-2 \text{ rect } (f/B)) \qquad (12)$$

and

$$C_o(f) = C''(f)\, C^*(f)$$

$$= 2\, C(f)\, C^*(f)\,[1-\text{rect}(f/B)]$$

$$= 2|C(f)|^2[1-\text{rect}(f/B)] \qquad (13)$$

Looking at Eq (13) it is observed that $|C(f)|^2$ is two times the energy spectral density of the spread spectrum code, S(f). Therefore, substituting Eq (3) into the above yields,

$$C_o(f) = 4Et_s \text{ sinc}^2(t_s f)\,[1-\text{rect } (f/B)] \qquad (14)$$

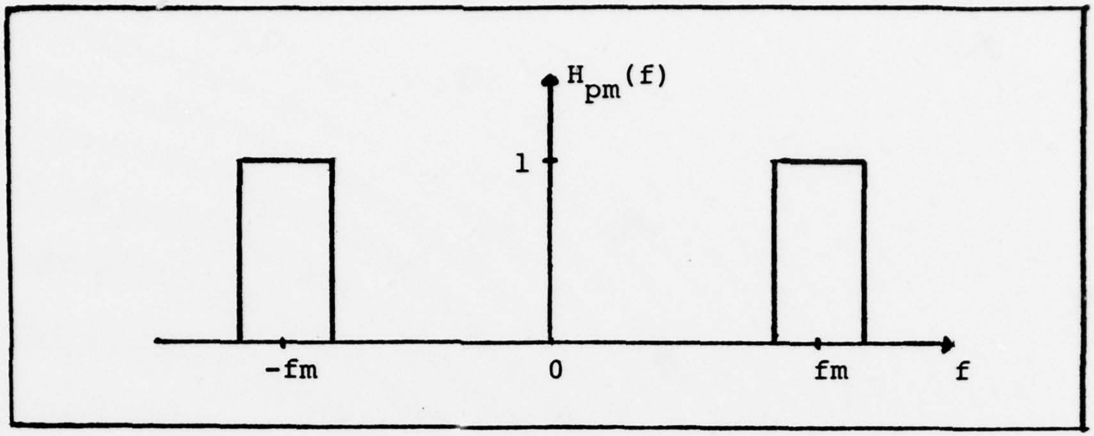


Figure 6. Plot of $H_{pm}(f)$ centered on fm.

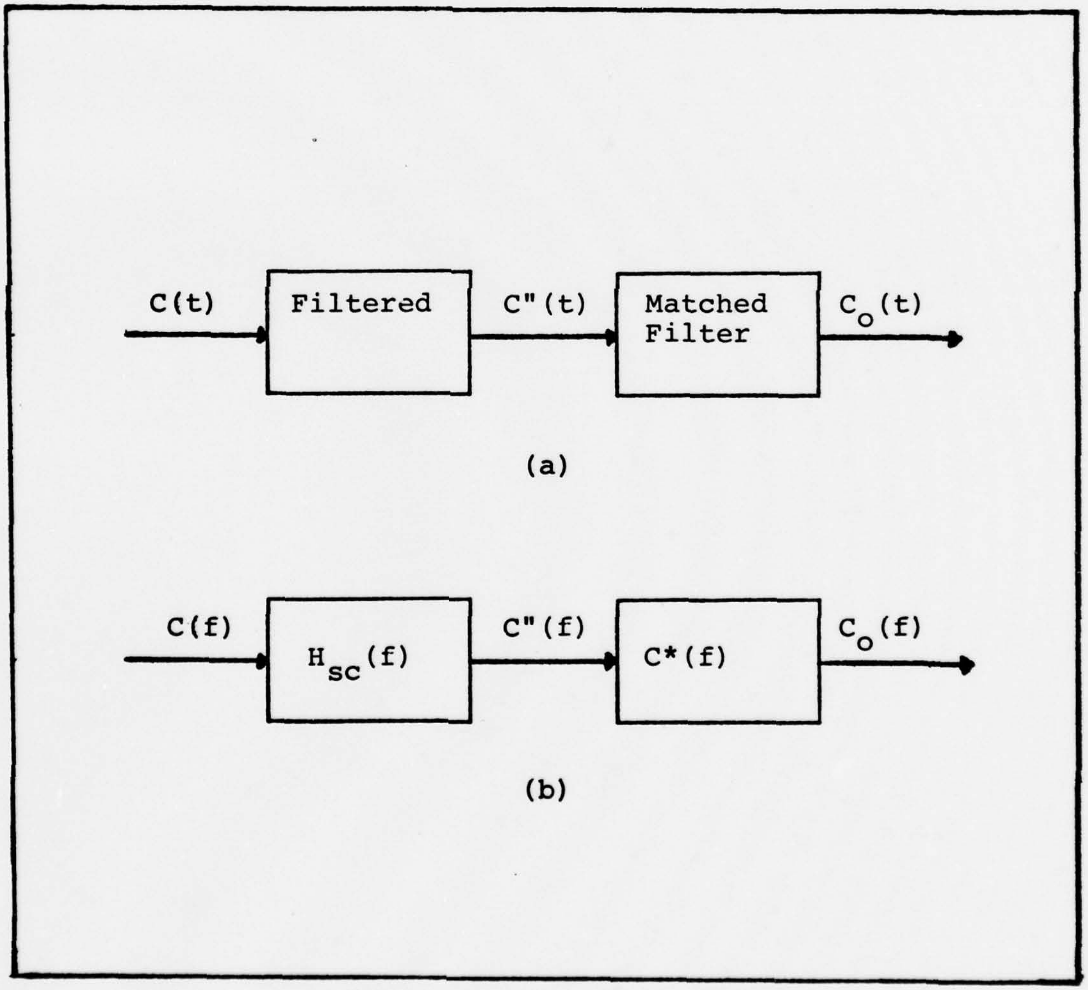


Figure 7. Baseband model of colored-noise filter and matched filter, (a) time domain and (b) frequency domain.

Taking the inverse Fourier transform of Eq (14) yields

$$\begin{aligned}
C_o(t) &= F^{-1}[C_o(f)] \\
&= F^{-1}[4Et_s \operatorname{sinc}^2(t_s f)[1-\operatorname{rect}(f/B)]] \\
&= 4EF^{-1}[t_s \operatorname{sinc}^2(t_s f) - t_s \operatorname{sinc}^2(t_s f) \\
&\qquad \operatorname{rect}(f/B)] \\
&= 4EF^{-1}[t_s \operatorname{sinc}^2(t_s f)] - 4EF^{-1}[t_s \operatorname{sinc}^2 \\
&\qquad (t_s f) \operatorname{rect}(f/B)] \\
&= 4E(1-|\tau|/t_s) \operatorname{rect}(\tau/2t_s) - 4Et_s \\
&\qquad F^{-1}[\operatorname{sinc}^2(t_s f) \operatorname{rect}(f/B)] \qquad (15)
\end{aligned}$$

The assumption that the bandwidth of the colored-noise filter is much smaller then the bandwidth of the spread spectrum signal ($B \ll W$, where W is equal to two divided by t_s) allows the following approximation:

$$1 \simeq \operatorname{sinc}^2(t_s f) \qquad -B/2 \le f \le B/2 \qquad (16)$$

When B is small enough compared to W, the magnitude of the sinc squared function remains within ±10 percent of its magnitude at the center of the colored-noise filter's bandwidth. Therefore,

$$\begin{aligned}
c_o(t) &= 4E(1-|t|/t_s) \operatorname{rect}(t/2t_s) - 4E\, t_s\, F^{-1}[\operatorname{rect}(f/B)] \\
&= 4E(1-|t|/t_s) \operatorname{rect}(t/2t_s) - 4Et_s B \operatorname{sinc}(Bt) \qquad (17)
\end{aligned}$$

When t is set equal to zero, the amount of degradation of the autocorrelation peak can be obtained. This sets the value of the triangle function and sinc(Bt) equal to one. Thus

$$c_o(0) = 4E = 4EBt_s \quad (18)$$

Dividing through by 4E in Eq (18) yields

$$c_o(0) = 1-Bt_s \quad (19)$$

The amount of normalized degradation caused by the colored-noise filter is now defined as

$$\begin{aligned} d &= Bt_s \\ &= 2(\frac{B}{W}) \end{aligned} \quad (20)$$

This assumes that B is small enough compared to W that the sinc squared function can be considered flat (±10%) over the bandwidth B. With this information, it is now possible to consider a direct sequence spread spectrum signal with bandwidth W. If t_s is defined to be the pulse width of the direct sequence code and f_s as the frequency at which the first zero of $\mathrm{sinc}^2\,(t_s f)$ occurs, then f_s equals W/2 and t_s is equal to 2/W. A plot of the frequency and time domains of the spread spectrum signal and the narrowband colored-noise filter are shown in Figures 8 and 9.

From Figures 8 and 9, we can observe that the same amount of peak degradation of the autocorrelation function is equal to the invers Fourier transform of the product of Figures 8a and 9a evaluated at t equal to zero, yielding $d-2ABt_s$. Because our assumption is good only for values of B that allow the power spectrum function to remain

within 10 percent of being flat, we will attach to each value of B a corresponding value of percent deviation (P).

Table I (Appendix A) contains the percent value of B divided by W, the amount of normalized peak deviation, the amount of signal power lost, and the percent deviation for each value of B/W. A direct sequence modulated spread spectrum system is being considered. The calculations for these values are shown in Appendix A. Figure 10 is a plot of 20 log (1-d), L, versus percent W. It is noted that for a sacrifice of 1 dB in the autocorrelation peak, incurred by a colored-noise filter with a bandwidth of 3 percent of the spread spectrum's code bandwidth, an attenuation of 25 dB in jamming signal strength can be achieved.

For B/W>8.75 percent, the amount of degradation to the triangular autocorrelation function is not constant over the interval $(-t_s, t_s)$. This is because the sinc(Bt) function over this interval can no longer be considered flat. Above this value of B/W, a larger degradation of the autocorrelation peak (t=0) then any other point in the interval is observed.

The assumption that the spread spectrum power density function remains within ten percent of the magnitude at the frequency corresponding to the filter's center frequency no longer holds above B/W equal to 8.75 percent. Therefore, our calculation of peak-degradation can no longer be considered accurate above this value for B/W.

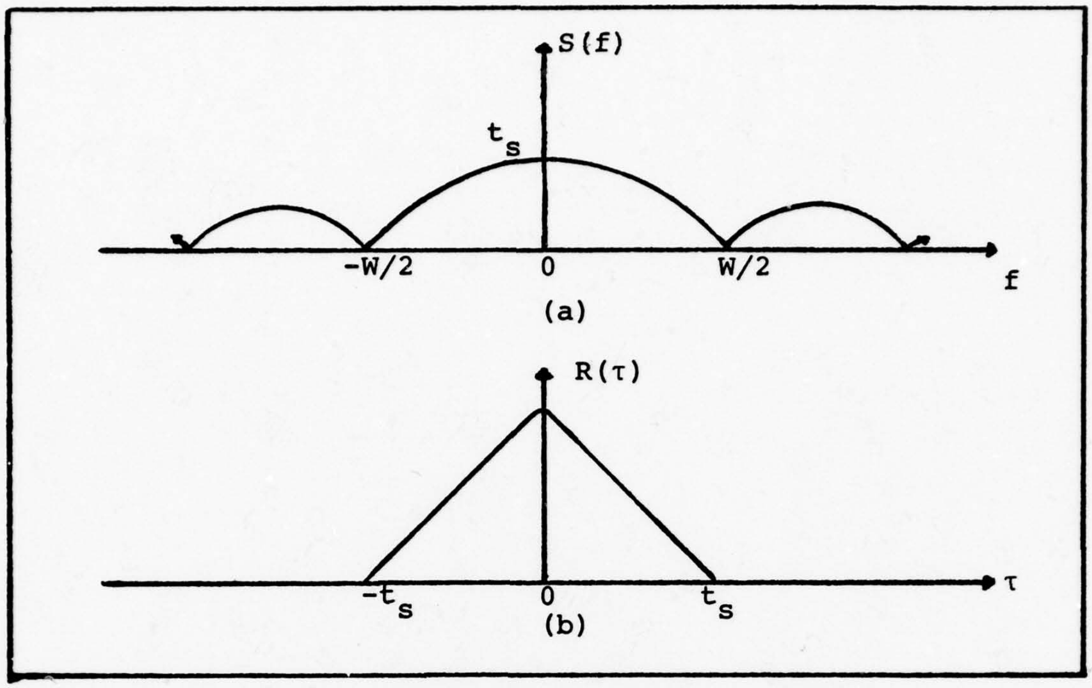


Figure 8. Normalized plot of (a) S(f); and, (b) inverse Fourier transform of S(f).

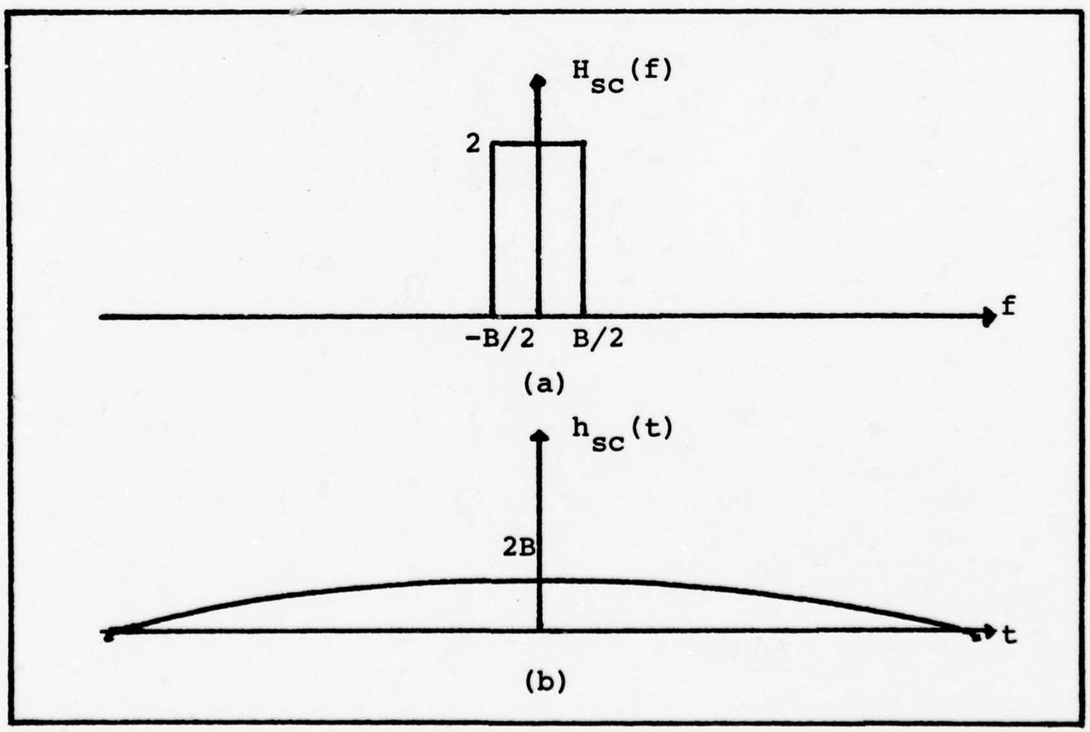


Figure 9. Baseband representation of (a) colored-noise filter, $H_{sc}(f)$; and, (b) inverse Fourier transform of $H_{sc}(f)$.

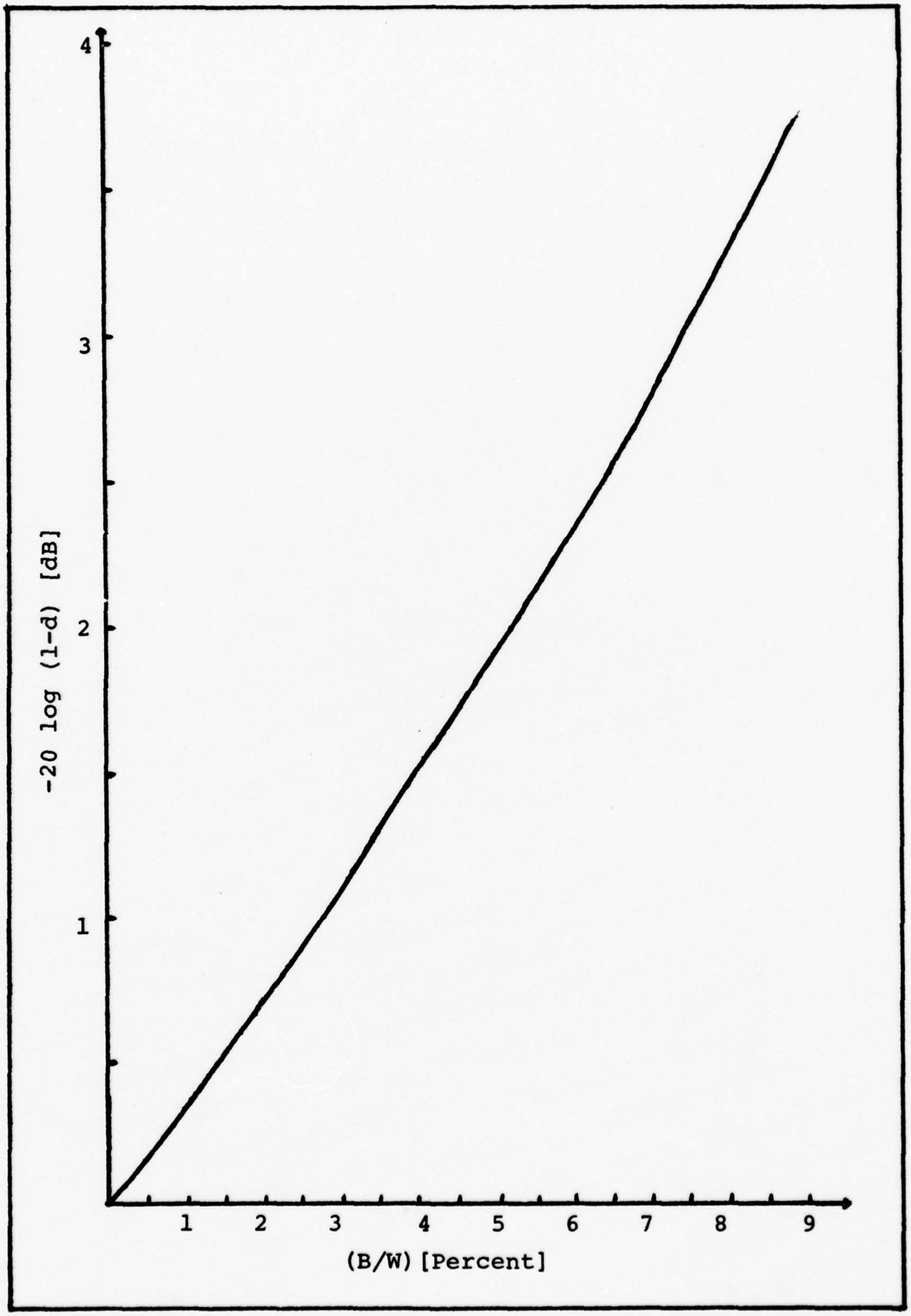


Figure 10. Plot of peak degradation versus percent B/W (filter centered)

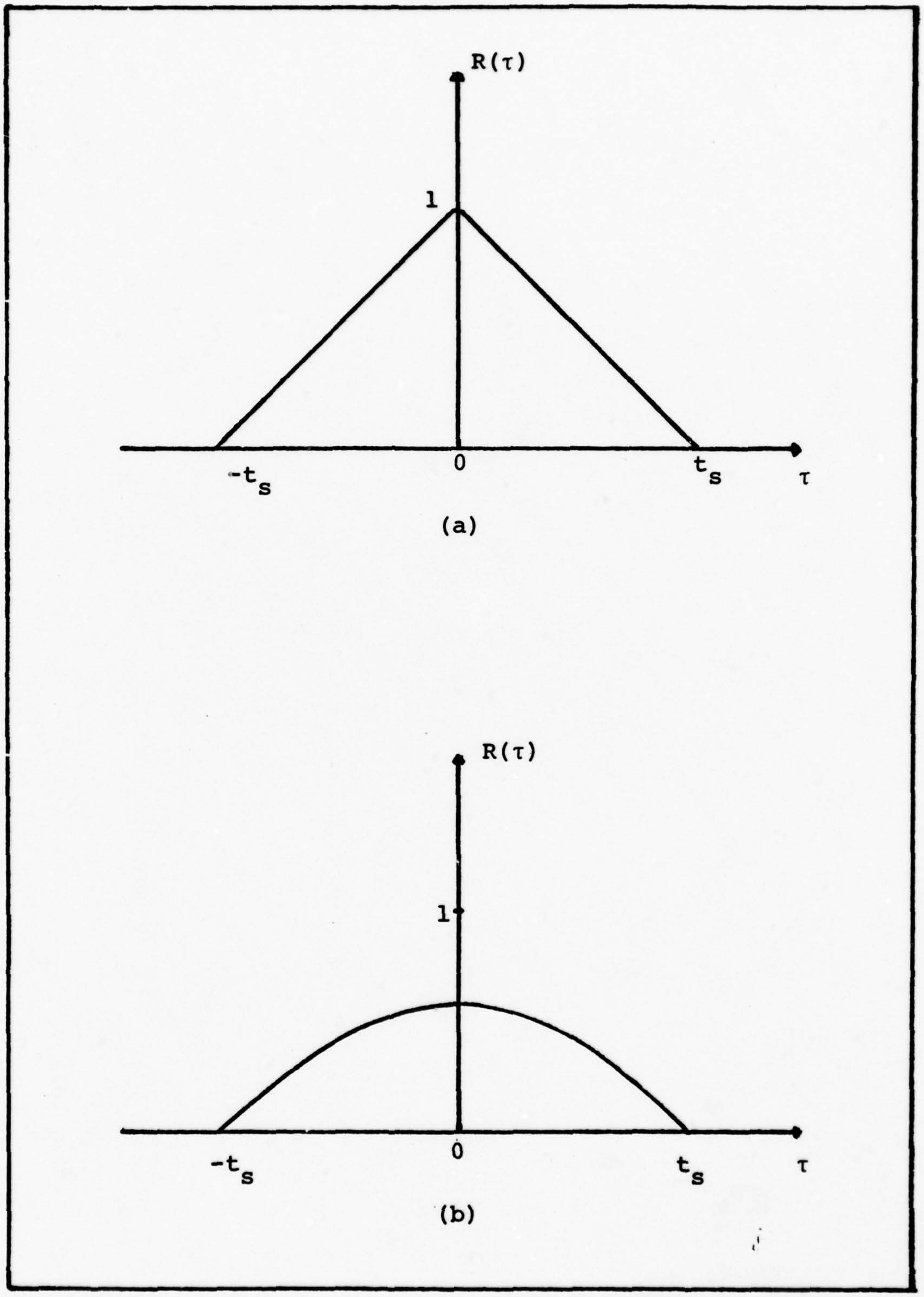


Figure 11. Spread Spectrum Autocorrelation Function (a) without colored-noise filter; and, (b) with colored-noise filter

Filter Offset by Fifty Percent from Center of Spread Spectrum Signal

In order to investigate the effects of a narrow pass filter with a center frequency shifted by an amount Δf Hz, attention is given to Figure 12, where Δf is equal to f_o minus f_m. Taking the inverse Fourier transform of the filter, we get the following:

$$h(t) = F^{-1}[\mathrm{rect}(\frac{f-f_o}{B}) + \mathrm{rect}(\frac{f+f_o}{B})]$$

$$= F^{-1}[\mathrm{rect}(\frac{f}{B})]\cdot(e^{+jw_o t} + e^{-jw_o t})$$

$$= 2B\ \mathrm{sinc}(Bt)\ \cos(w_o t)$$

$$= \mathrm{Re}(2B\ \mathrm{sinc}(Bt)\ (\cos(w_o t) + j\ \sin(w_o t)))$$

$$= \mathrm{Re}(2B\ \mathrm{sinc}(Bt)\cdot e^{jw_o t}) \qquad (21)$$

where

w_o is $2\pi f_o$

H(f), the Fourier transform of h(t), is a single square pulse centered at f=0. In order to shift it to the desired location, the following is performed

$$h(t) = \mathrm{Re}\ (2B\ \mathrm{sinc}(Bt)\ e^{+j(w_o-w_m)t}\ \ e^{+jw_m t}) \qquad (22)$$

where

w_m is $2\pi f_m$

Defining h(t), the baseband representation of the colored-noise filter as

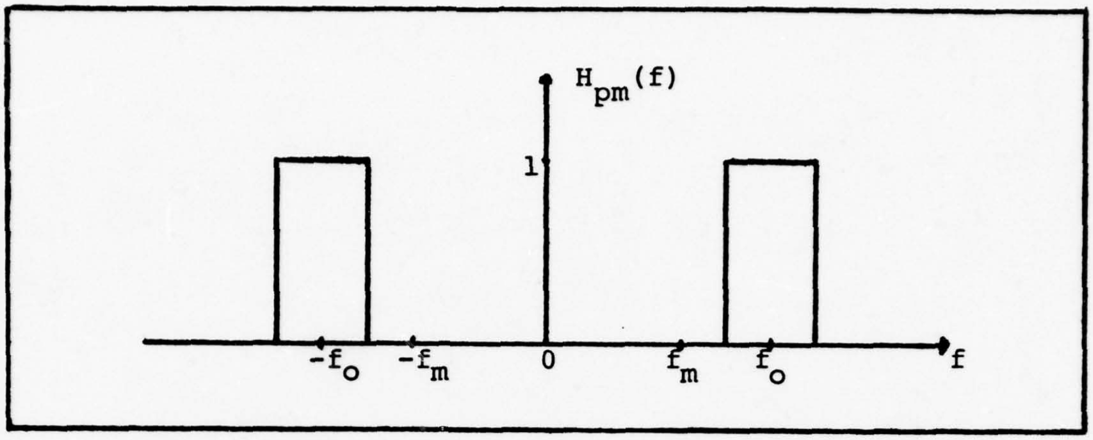


Figure 12. Colored-noise filter offset from f_m.

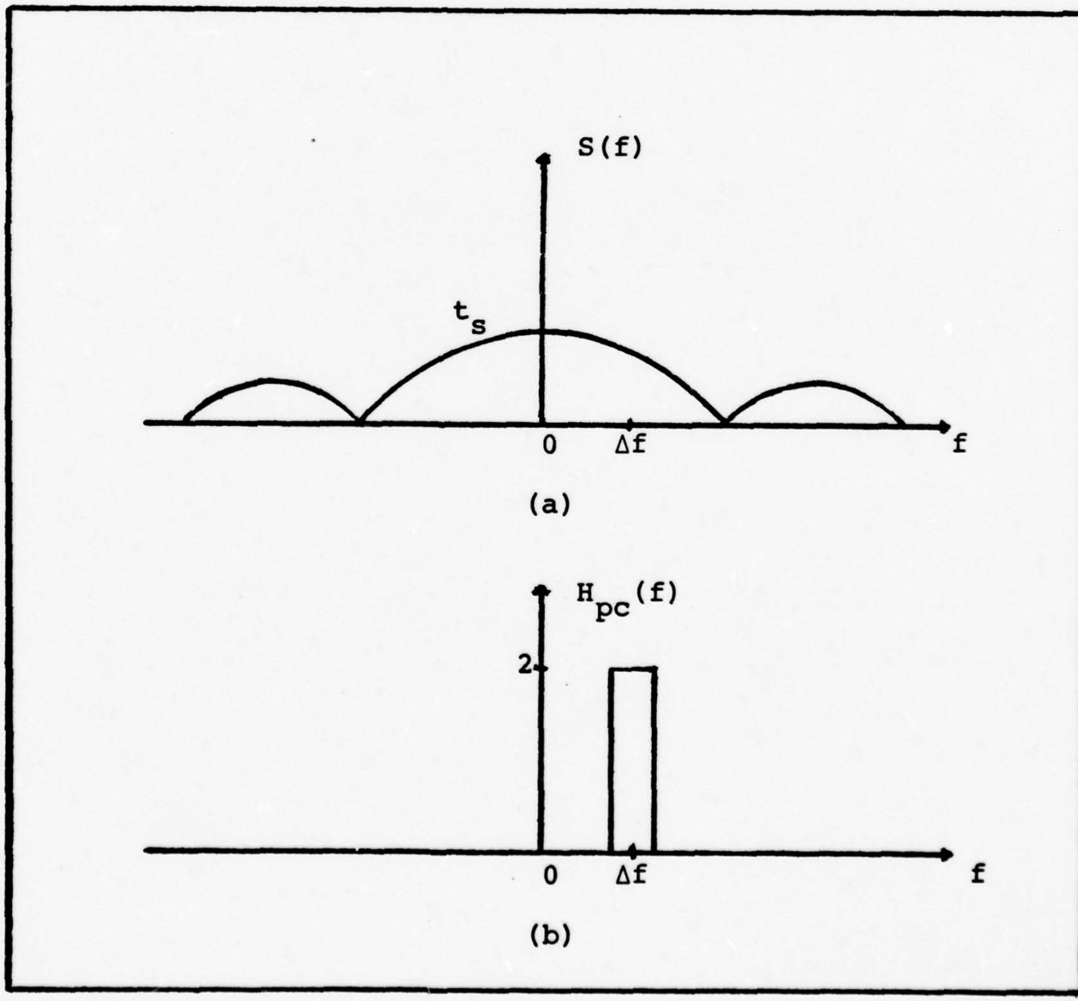


Figure 13. Plot of (a) spread spectrum power density function; and, (b) narrowband colored-noise filter centered on Δf.

$$h_{pc}(t) \triangleq (2B)\ \mathrm{sinc}(Bt)\ e^{+j(w_o - w_m)t} \tag{23}$$

and using,

$$\Delta w = w_o - w_m \tag{24}$$

results in

$$h_{pc}(t) = 2B\ \mathrm{sinc}(Bt)\ e^{+j\Delta wt} \tag{25}$$

Taking the Fourier transform, yields

$$H_{pc}(f) = 2\ \mathrm{rect}\left(\frac{f-\Delta f}{B}\right) \tag{26}$$

Substituting Eq (26) into Eq (14) results in

$$C_o(f) = 4Et_s\ \mathrm{sinc}^2(t_s f)\left(1-\mathrm{rect}\left(\frac{f-\Delta f}{B}\right)\right) \tag{27}$$

Defining the amount of degradation introduced by the filter as

$$d(t) = F^{-1}\left(4Et_s\ \mathrm{sinc}^2(t_s \Delta f)\cdot \mathrm{rect}\left(\frac{f-\Delta f}{B}\right)\right) \tag{28}$$

Because of the assumption that the power spectral function is considered flat (±10%) over the filter's bandwidth, the sinc squared term can be treated as a constant. Therefore,

$$\begin{aligned} d(t) &= 4Et_s\ \mathrm{sinc}^2(t_s\Delta f)\ F^{-1}\left(\mathrm{rect}\left(\frac{f-\Delta f}{B}\right)\right) \\ &= 4Et_s\ \mathrm{sinc}^2(t_s\Delta f)\ B\ \mathrm{sinc}(Bt)\ e^{+j\Delta wt} \\ &= 4EBt_s\ \mathrm{sinc}^2(t_s\Delta f)\ \mathrm{sinc}(Bt)\ e^{+j\Delta wt} \end{aligned} \tag{29}$$

At this point, the only concern is with the amount of degradation of the correlation peak; therefore, t is again

set equal to zero and Eq (29) becomes

$$d(0) \quad = \quad 4EBt_s \ \text{sinc}^2(t_s\Delta f) \qquad (30)$$

Dividing Eq (30) by 4E, as was done with Eq (18), the normalized degradation caused by the filter is given by

$$\begin{aligned} d \quad &= \quad Bt_s \ \text{sinc}^2(t_s\Delta f) \\ &= \quad \frac{2B}{W} \ \text{sinc}^2\left(\frac{\Delta f}{W/2}\right) \end{aligned} \qquad (31)$$

Since the filter is now centered on a frequency equal to Δf, a Δf now must be chosen to observe the effects of shifting the filter. Again, the amplitude of the spread spectrum power density function is considered to be constant, within 10% of its magnitude at Δf, over the bandwidth of the filter. This holds true only for $B \ll W$. Using the example of the previous section and choosing Δf equal to W/4 places the filter midway between the peak of the spread spectrum's power spectrum function and its first null.

Table II gives the percent of B divided by W, the amount of normalized peak degradation (d), the amount of signal power lost (L), and the percent deviation (P) of the spread spectrum density function from being at its amplitutde at frequency Δf for each value of B/W (see Appendix B). Using the same direct sequence modulated spread spectrum system as was used in the previous section, the calculations for values of B/W are shown in Appendix B. Figure 11 is a plot of L, equal to 20 log (1-d), versus

percent B/W.

The assumption that the power density function remains flat (±10%) over the bandwidth of the filter is valid up to B/W equal to 12.5 percent. This is a substantial decrease in the filter's bandwidth where the assumption remains valid, compared to the non-shifted case. This is because the filter is positioned on a relatively steeper portion of the $t_s \operatorname{sinc}^2(t_s \Delta f)$ curve.

Appendix C shows the calculations for an upper bound on the amount of autocorrelation peak degradation due to the filter. The value for d is computed using the largest magnitude of the power spectral function contained within the filter's frequency band. This corresponds to using the magnitude at frequency f_L. Table III (Appendix C) shows the results and Figure 11 gives a plot of L versus percent W. The values for B/W and P are the same for corresponding values of B in Table II.

Appendix D shows the calculations for the lower bound on the amount of autocorrelation peak degradation due to the colored-noise filter. The value for d, this time, is computed using the smallest magnitude of the power spectral function contained within the filter's frequency band. This corresponds to using the value at frequency f_H. Table IV shows the results of these calculations and Figure 14 gives a plot of L versus percent W.

This chapter has considered the case when the colored-

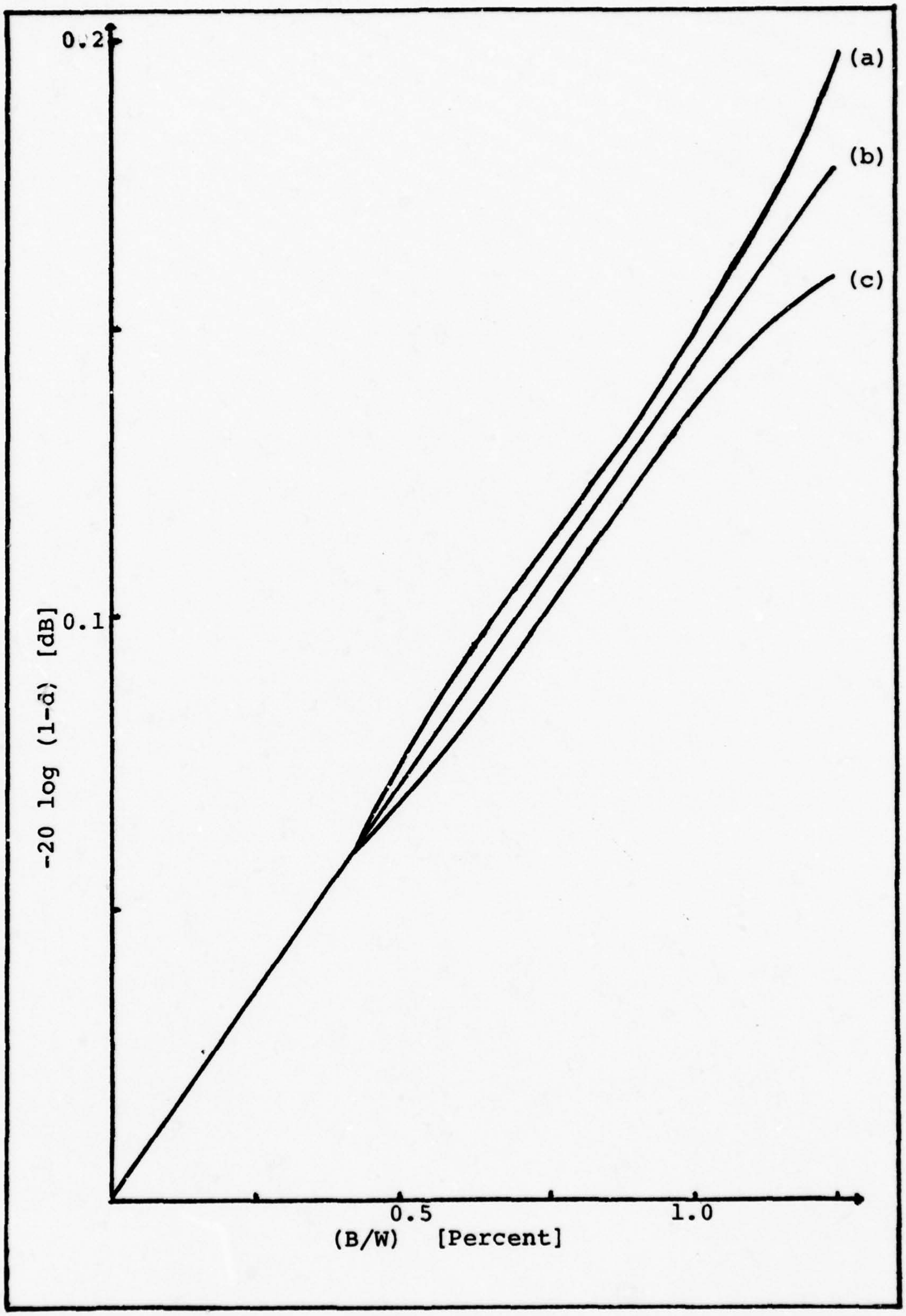


Figure 14. Plot of peak degradation versus percent B/W (filter offset) (a) upper bound; (b) average, and (c) lower bound.

noise filter is centered on the spread spectrum signal and when the filter is offset by an amount equal to W/4. The amount of desired signal autocorrelation peak degradation is given by Eqs(20) and (30) for the centered and offset cases respectively. Tables I and II (Appendix A and B, respectively) show some computed values using these equations for various values of B/W. Comparing Tables I and II, it is seen that the greatest amount of autocorrelation peak degradation for a given value of B/W is when the filter is centered on the signal. From Eq (31) it can be seen that the larger the offset the less effect the filter has on the autocorrelation peak. However, the larger the filter's bandwidth, the larger the amount of degradation. Thus the first criterion indicates that a colored-noise filter can improve performance in an adaptive spread spectrum communications system being jammed by its largest threat: a narrowband jammer (see Appendix F). This leads to the next chapter which looks at the second criterion to be investigated: signal-to-noise improvement.

III. Signal-to-Noise Ratio

In order to evaluate the performance improvement by introducing a colored-noise filter into a spread spectrum communication's receiver, a comparison of the signal-to noise ratios of the receiver with and without the colored-noise filter is made. Section I contains the analysis for the signal-to-noise ratio of the receiver without the filter. Section II does the same analysis for the receiver with the filter. In both analyses, the spread spectrum signal is considered to be a binary information stream modualted by a direct sequence code with a total energy given by Eq (2) and has a bandwidth W. The noise of the receiver is considered to have a flat spectral density over the bandwidth W. Lastly, the jamming signal is also considered to have a flat spectral density over its bandwidth B.

Spread Spectrum Receiver Without a Colored-Noise Filter

In a coherent spread spectrum communications system, the received signal, r(t), with colored-noise (interference jamming) can be represented at baseband as:

$$r(t) = i(t)c(t) + n(t) + j(t) \tag{32}$$

where

$i(t)$	is the information signal (equal to ± 1)
$c(t)$	is the spread spectrum code at baseband
$n(t)$	is bandlimited white noise

$j(t)$ is the colored-noise (interference)

All signals are represented by their complex envelopes. Figure 15 shows the correlation implementation of the spread spectrum with $c^*(t)$ being equal to the complex conjugate of $C(t)$. From Figure 15 and Eq (3), we have:

$$\begin{aligned} y(T) &= \int_0^T [c(t)C(t) + n(t) + j(t)]c^*(t)\,dt \\ &= \int_0^T c(t)C(t)c^*(t)dt + \int_0^T n(t)c^*(t)dt + \\ &\qquad \int_0^T j(t)c^*(t)dt \end{aligned} \tag{33}$$

The output of the correlator, $y(T)$, at time t equal to T is a number with units of volts, thus, $c^*(t)$ is unitless. On a per ohm basis the output power, P_c, due to the spread spectrum signal is given by

$$P_c = [y_c(T)]^2 \tag{34}$$

where

$$\begin{aligned} y_c(T) &= \int_0^T i(t)C(t)c^*(t)dt \\ &= \int_0^T i(t)|c(t)|^2 dt \end{aligned} \tag{35}$$

Therefore,

$$P_c = [\int_0^T |C(t)|^2 dt]^2 \tag{36}$$

Since by definition $i(t)$ is either plus or minus one, then $i(t)$ squared is equal to plus one. Using Eqs (E-10) and (E-11)

$$P_c = 4E^2$$

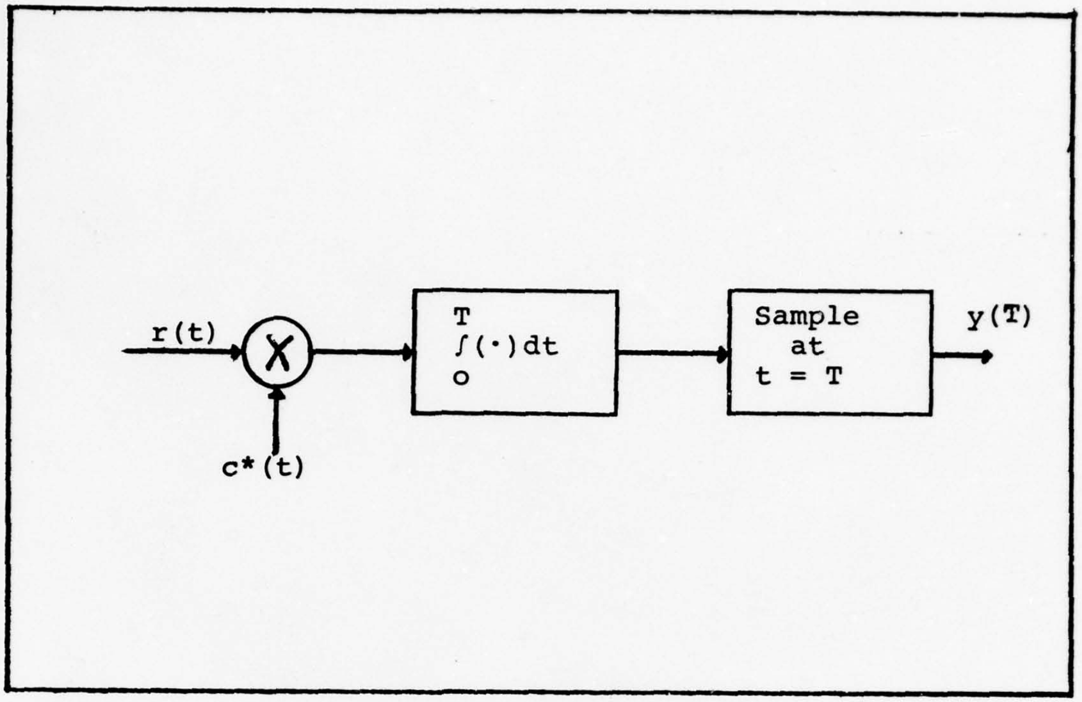


Figure 15. Correlation receiver.

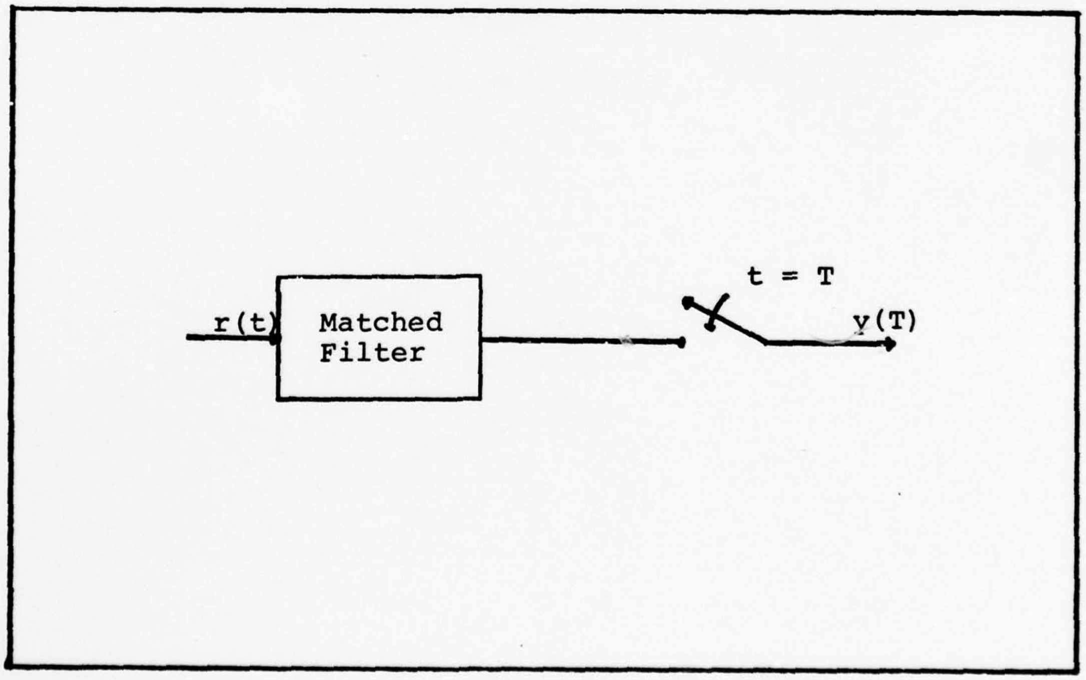


Figure 16. Matched filter receiver.

To compute the output power due to the noise, the receiver is represented as a matched filter in Figure 16. This is allowed since a matched filter receiver is equivalent to a correlator receiver at sampling instant T.

The matched filter is matched to the received spread spectrum signal. Thus, if the impulse response of the filter is defined as h(t) and H(f) as the Fourier transform of h(t), then

$$H(f) = C^*(f) \tag{38}$$

where

$C^*(f)$ is the Fourier transform of $c^*(t)$

White noise with a power spectral density $S_{in} = \frac{N_o}{2}$, at the input of this filter, results in an output noise with power spectral density $S_{on}(f)$ where

$$\begin{aligned} S_{on}(f) &= S_{in}|H(f)|^2 \\ &= \frac{N_o}{2}|C(f)|^2 \end{aligned} \tag{39}$$

Integrating S_{on} over all frequencies gives us the total output noise power, thus,

$$\begin{aligned} P_n &= \int_{-\infty}^{\infty} \frac{N_o}{2}|C(f)|^2 \, df \\ &= \frac{N_o}{2}\int_{-\infty}^{\infty}|C(f)|^2 \, df \\ &= N_o E \end{aligned} \tag{40}$$

Eq (40) makes use of Eq (E-9).

Turning to the colored-noise (jammer), Figure 13 is again used with the matched filter matched to the received desired spread spectrum signal c(t). The colored noise has a power spectal density $S_{ij} = J_o/2$ over its bandwidth B.

The jammer's bandwidth is assumed much smaller than the bandwidth of the spread spectrum signal ($B<<W$).

The spread spectrum code appears as in Figure 3, and has an autocorrelation function, $R(\tau)$, given in Eq (3) and shown in Figure 4. Being concerned with baseband representation and using the ideal (one-shot correlation) case the total energy is given by Eq (2) as

$$E = A^2 N t_s / 2 \tag{2}$$

Taking the Fourier transform of $R(\tau)$ yielded the energy spectral density, $S(f)$, given by Eq (3) as

$$S(f) = E t_s \operatorname{sinc}^2(t_s f) \tag{3}$$

Using the equality

$$W = 2/t_s \tag{41}$$

where

W is the bandwidth of the spread spectrum signal

t_s is the pulse width of one spread spectrum chip

Eq (3) becomes

$$S(f) = 2(E/W), \operatorname{sinc}^2(t_s f) \tag{42}$$

The jammer has a power spectrum $S_j(f)$ whose magnitude is considered constant and equal to $J_o/2$ for $-B/2 \leq f \leq B/2$, and zero elsewhere. (Again, B is much less then W).

The output of the matched filter of Figure 13 is the

noise energy density S_{oj}, where

$$S_{oj}(f) = S_j(f)\ |H(f)|^2$$
$$= S_j(f)\ |C(f)|^2$$

To obtain the total output colored-noise power, P_j, Eq (43) is integrated over all frequencies.

$$P_j = \int_{-\infty}^{\infty} S_{ij}(f)\,|C(f)|^2 df$$
$$= \int_{-\infty}^{\infty} 2S_{ij}(f)\ S(f)\ df \qquad (44)$$

Substitution of Eq (42) into Eq (44) yields

$$P_j = \int_{-B/2}^{B/2} \frac{2J_oE}{W}\ \mathrm{sinc}^2\ (t_sf)\ df$$
$$= \frac{2J_oE}{W} \int_{-B/2}^{B/2} \mathrm{sinc}^2\ (t_sf)\ df \qquad (45)$$

where

$$\int_{-B/2}^{B/2} \mathrm{sinc}^2\ (t_sf)\ df \simeq B \qquad (46)$$

since

$$\mathrm{sinc}^2\ (t_sf) = 1 \qquad \text{when } B \ll \frac{1}{t_s} \qquad (47)$$

Therefore,

$$P_j = \frac{2EJ_oB}{W} \qquad (48)$$

The signal-to-noise ratio at the output of the correlator, without a notch filter, is

$$SNR_{w/o} = \frac{P_c}{P_n + P_j}$$

$$= \frac{E^2}{N_o E + \frac{2J_o BE}{W}}$$

$$= \frac{E}{N_o + \frac{2J_o B}{W}} \tag{49}$$

Eq (49) gives the signal-to-noise ratio without the use of a colored-noise filter. It is now necessary to calculate the signal-to-noise ratio of the same system but with a colored-noise filter. Once this is done, then signal-to-noise ratio with the filter can be divided by the signal-to-noise ratio without the filter and the result compared to one. When the resulting equation is greater than one, a signal-to-noise ratio improvement is possible. The next section does the analysis with the filter.

Spread Spectrum Receiver with a Colored-Noise Filter

The introduction of an ideal colored-noise filter into the process so that the colored-noise (jammer is completely removed yields the receiver block diagram of Figure 17, where $h_j(t)$ is the impulse response of the colored-noise filter and $H_j(f)$ is its Fourier transform. The output $C_o(f)$, where the transfer function $H_j(f)$ is that of an ideal bandstop filter centered on zero, is

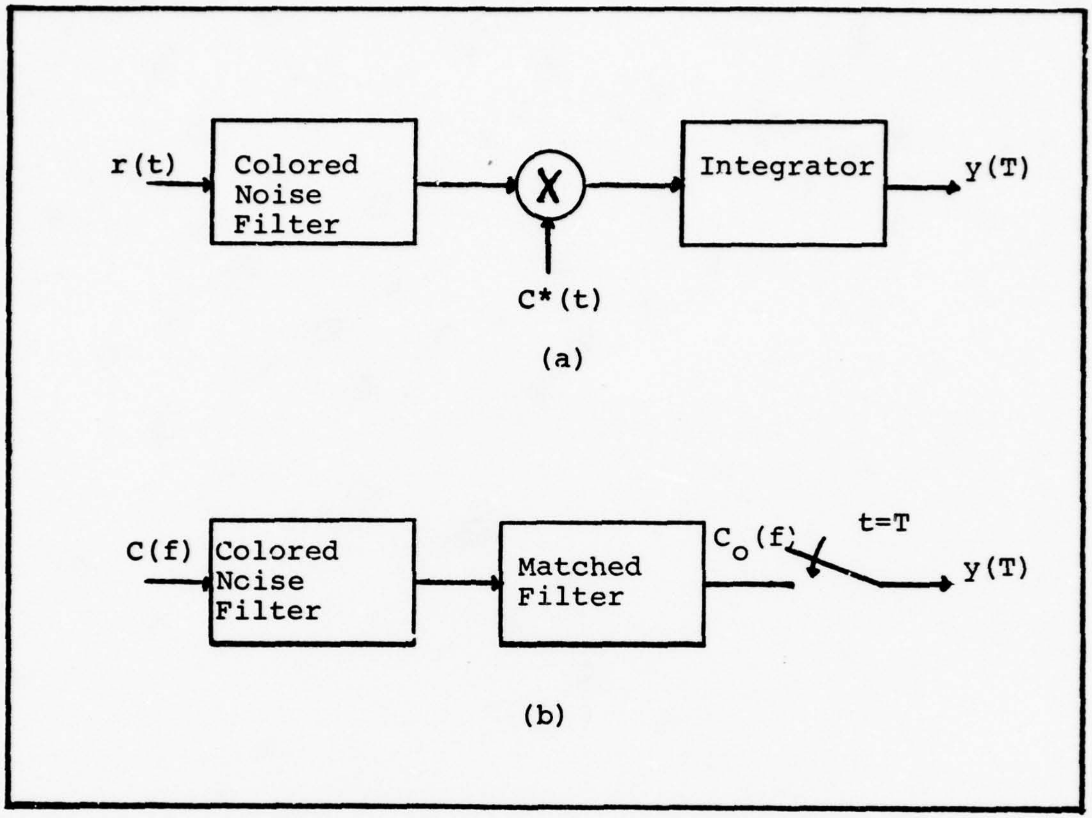


Figure 17. Receiver with colored-noise filter (a) correlator representation; and, (b) matched filter representation

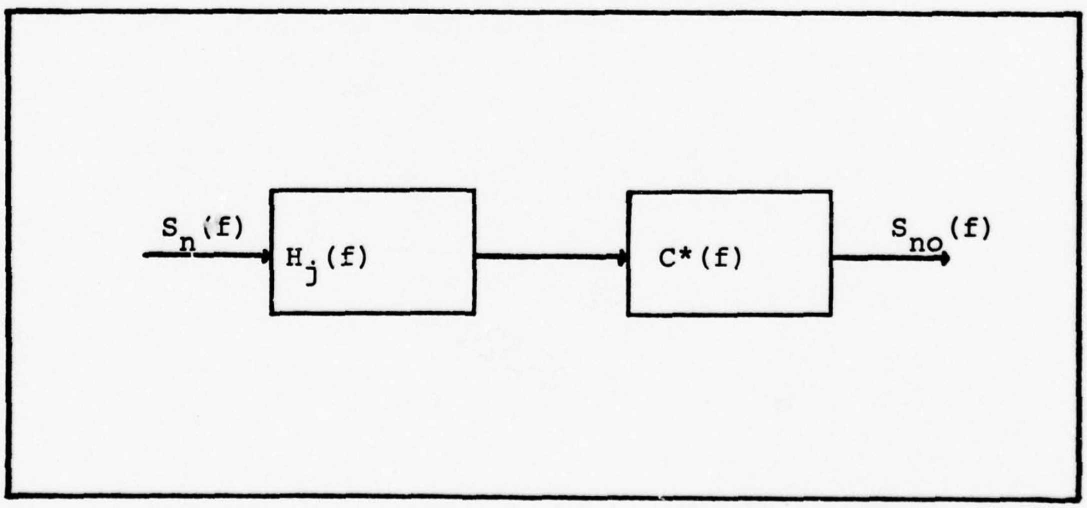


Figure 18. Receiver block diagram with colored-noise filter for noise power calculations

given by Eq (13),

$$C_o(f) = 2|C(f)|^2 [1-H_{1p}(f)] \tag{13}$$

where $H_j(f)$ is equal to $H_{sm}(f)$ of Eq (5). Following the same procedure as was done for Eqs (14) through (18) yields for $C_o(t)$ evaluated at t equal to zero,

$$\begin{aligned} C_o(0) &= 4E - 4EBt_s \\ &= 4E - 8E(B/W) \end{aligned} \tag{50}$$

where t_s is equal to two divided by the bandwidth (W) of the spread spectrum signal. Manipulating the above to obtain:

$$\begin{aligned} C_o(0) &= 4E - 8E(B/W) \\ &= (4E/W)W - (4E/W)2B \\ &= (4E/W)(W-2B) \end{aligned} \tag{52}$$

The output noise power with the colored-noise filter is calculated using Figure 18 where $S_n(f)$ is the power spectral density of the noise.

$$\begin{aligned} S_{no}(f) &= S_n(f)\ |H_j(f)|^2\ |C^*(f)|^2 \\ &= (1/2)N_o|C(f)|^2\ |H_j(f)|^2 \end{aligned} \tag{53}$$

Integrating over all frequencies to obtain the total noise power yields

$$\begin{aligned} P_n &= \int_{-\infty}^{\infty} (1/2)N_o|C(f)|^2\ |H_j(f)|^2\ df \\ &= (1/2)N_o \int_{-\infty}^{\infty} |C(f)|^2\ |H_j(f)|^2\ df \\ &= (1/2)N_o \int_{-\infty}^{\infty} 4|C(f)|^2\ |1-H_{1p}(f)|^2\ df \end{aligned} \tag{54}$$

where

$$[1-H_{1p}(f)]^2 = 1 - H_{1p}(f) \tag{55}$$

Therefore

$$P_n = (1/2)N_o \int_{-\infty}^{\infty} 4[|C(f)|^2 - |C(f)|^2 |H_{1p}(f)|^2]df$$

$$= 2N_o [2E - \int_{-B/2}^{B/2} |C(f)|^2 df$$

$$= 2N_o [2E-E \frac{4B}{W}]$$

$$= 2N_o 2E[\frac{W}{W} - \frac{2B}{W}]$$

$$= N_o \frac{4E}{W} (W-2B) \tag{56}$$

The signal-to-noise ratio with the colored-noise filter is,

$$SNR_w = \frac{P_c}{P_n}$$

$$= \frac{(\frac{4E}{W})^2 (W-2B)^2}{N_o \frac{4E}{W} (W-2B)}$$

$$= \frac{4E}{N_o W} (W-2B) \tag{57}$$

The amount of improvement gained by using the colored-noise filter can be expressed as the ratio of the SNR with the filter and the SNR without the filter.

$$\frac{SNR_w}{SNR_{w/o}} = \frac{\frac{4E}{N_o W} (W-2B)}{\frac{4E}{N_o \frac{2J_o B}{W}}}$$

$$= \frac{(W-2W)}{N_o W} \quad [N_o + \frac{J_o B}{W}]$$

$$= \frac{W-2B}{w} \quad (1 + \frac{2J_o B}{N_o W}$$

$$= (1 - \frac{2B}{W}) \quad (1 + \frac{J_o 2B}{N_o W} \tag{58}$$

Thus, the amount of improvement depends on two ratios or parameters. The first is the jammer-to-signal bandwidth ratio and the second is the jammer-to-noise power ratio. An improvement capability exists whenever Eq (58) is greater than one, that is when

$$\frac{J_o}{N_o} > \frac{1}{1 - \frac{2B}{W}} \tag{59}$$

or equivalently,

$$\frac{2B}{W} < 1 - \frac{N_o}{J_o} \tag{60}$$

IV. Intersymbol Interference

Intersymbol interference occurs when the response of one or more symbols is still present when the current symbol is being processed. In the case of the spread spectrum system, the addition of a colored-noise filter with a narrow bandwidth results in a very long time response compared to that of the autocorrelation. If the response of the filter is longer than the interval between the autocorrelation peaks, then intersymbol interference will occur. The interval between autocorrelation peaks is a function of Nt_s (code period).

If the time of the current autocorrelation peak is denoted as t_0 and that of the previous autocorrelation peak as t_1, then

$$t_o - t_1 = Nt_s \tag{61a}$$

In general

$$t_o - t_k = kNt_s \qquad k=0,1,2,\ldots \tag{61b}$$

The effect of the filter on the autocorrelation function is given in Eq (17). Normalizing Eq (17) and using only the portion due to the filter (second term) yields

$$d(t) = t_s B \operatorname{sinc}(Bt) \tag{62}$$

where,

$d(t)$ is the degradation due to the colored-noise filter

B is the bandwidth of the colored-noise filter

Since only the peak degradation is of interest, the values for t given by Eq (61) are substituted into Eq(62) to yield

$$d = \sum_{k=0}^{\infty} t_s B \; \text{sinc} \; (kBNt_s) \tag{63}$$

where k equal to zero corresponds to the present filter's response associated with the present autocorrelation peak and k equal to one corresponds to the filter response associated with the previous autocorrelation peak, and so on. In any given system t_s and B will be fixed at any instant of time, therefore, for the degradation caused by intersymbol interference,

$$d_i = t_s B \sum_{k=1}^{\infty} \text{sinc} \; (kBNt_s) \tag{64}$$

The lower limit on k has changed from zero to one because setting k equal to zero yields Eq (20), which is not defined to be intersymbol interference. As long as our assumption that the colored-noise filter's bandwidth (B) is much smaller than the spread spectrum signal's bandwidth (W), then there will be constructive as well as destructive interference. From Eq (64), it can be seen that as k increases the magnitude of the sinc function envelope decreases and at infinity goes to zero, thereby having no effect on the current autocorrelation function peak. It is important to note that in Eq (64) the data is considered

to be all "plus ones". This is the worst case possible because when the data varies there will be more constructive interference then accounted for in Eq (64). Also, the filter is assumed to be centered on the spread spectrum signal, thus insuring a worst case consideration.

Because Eq (64) is an unbounded summation, it becomes necessary to approximate its value by means of numerical evaluation on a computer. This evaluation resulted in the graph of Figure 19, where the value for Nt_sB monotonically increases while k is held constant. Figure 19 shows that as Nt_sB increases the amount of intersymbol interference decreases exponentially. Therefore, the effects of intersymbol interference can be reduced by the use of a spread spectrum code with a maximal code length, assuming that B is constrained to a maximum value. It is observed that as Nt_s increases the period of the spread spectrum code is increased and the data is slowed down proportionately. This allows more energy for each data bit and thus increases jamming resistance. This assumes that the information bit contains one period of the spread spectrum code.

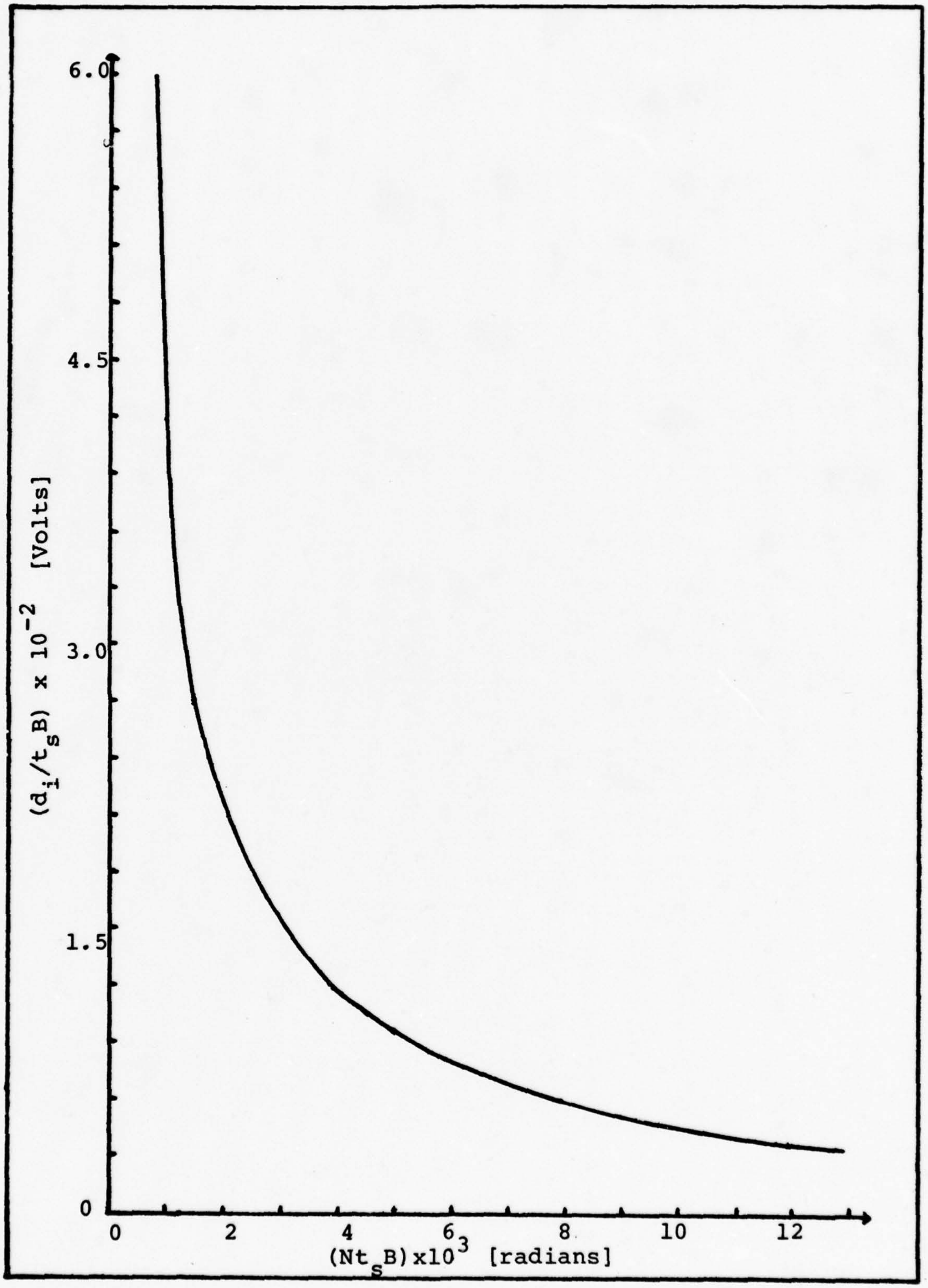


Figure 19. Plot of Eq (64) normalized by t_sB.

V. Conclusions

One effective method to improve performance in a communications system is the use of spread spectrum techniques. A second method is to adaptively filter out any interference signal before it is processed by the receiver. In an effort to achieve even greater performance, this thesis investigated the effects of combining both of these methods. It was shown in Appendix F that the greatest threat to a spread spectrum communications system is a narrowband jammer. In order to negate this threat, the use of a programmable adaptive tapped delay line to provide a narrow bandstop (colored-noise) filter could be used. The adaptive portion of the filter allows the filter to locate the narrowband jammer and adapt to its bandwidth. For the purpose of this thesis, the adaptive filter is considered to follow the jammer perfectly, thereby allowing for complete filtering.

To determine the effects of this colored-noise filter in a spread spectrum receiver, three different criteria were designed: autocorrelation peak degradation, signal-to-noise ratio, and intersymbol interference. The introduction of any type of filter causes a reduction in the amount of desired signal power which reaches the output of the receiver. Chapter II shows that for a jamming signal located within the spread spectrum bandwidth, the amount of autocorrelation peak degradation, d, is given by

Eq (31)

$$d = Bt_s \, \text{sinc}^2(t_s \Delta f) \tag{31}$$

where

B is the bandwidth of the jamming signal

t_s is the chip interval of the spread spectrum code

Δf is the amount of frequency offset of the center of the jamming signal from the center of the spread spectrum signal.

Thus, the amount of peak degradation is dependent on the jammer's bandwidth and relative position. The smaller the value of B, the smaller the degradation; and the larger the relative center frequency offset, the smaller the degradation. Thus, for the largest threat against a spread spectrum communications system (narrowband jamming) there is a relatively small amount of autocorrelation peak degradation (Figure 7).

The second criterion for performance was the comparison of signal-to-noise ratios of the receiver with and without the colored-noise filter. In both analyses, the spread spectrum signal is considered to be a binary information signal modulated by a direct sequence code with a fixed total energy, E, and a bandwidth, W. The noise of the receiver is considered to have a flat spectral density over the bandwidth W. Likewise, the jamming signal is considered to have a flat spectral density over its bandwidth B, where B is much smaller than W. Chapter III goes

through the analyses and yields the resulting equation

$$\frac{SNR_W}{SNR_{W/O}} = (1-\frac{2B}{W})(1+\frac{2J_O B}{N_O W}) \tag{58}$$

where

J_O is twice the baseband jammer power spectral density

N_O is twice the baseband noise power spectral density

An increased performance capability exists whenever Eq (58) yields a value greater than one. This occurs whenever

$$\frac{J_O}{N_O} > \frac{1}{1-2\frac{B}{W}} \tag{59}$$

or equivalently

$$2\frac{B}{W} < 1-\frac{N_O}{J_O} \tag{60}$$

The last criterion considered was the resulting intersymbol interference caused by the introduction of a colored-noise filter. Chapter IV presented the calculations yielding Eq (63) which gives the amount of degradation (d_i) due to intersymbol interference.

$$d_i = t_s B \sum_{k=1}^{\infty} \operatorname{sinc}(k\ BNt_s) \tag{63}$$

where

t_s is the chip interval of the spread spectrum code

B is the bandwidth of the colored-noise filter

N is the number of chips in the spread spectrum codeword

k is the number of autocorrelations considered

Eq (63) indicates that the amount of autocorrelation peak degradation due to intersymbol interference is determined by t_s, B, and N. If t_s and B were to be considered as fixed at any given time, then N would be the critical factor in Eq (63). This is shown in Figure 16.

Based on the three criteria of autocorrelation peak degradation, signal-to-noise ratio improvement factor, and intersymbol interference an increased anti-jam capability exists in an adaptive spread spectrum system. For a sacrifice of 1 dB in the autocorrelation peak, incurred by the introduction of a colored-noise filter with a bandwidth of 3 percent of the spread spectrum's code bandwidth, an attenuation of 25 dB in jamming power can be achieved. This increased anti-jam performance may one day make the difference between command and control communications and chaos.

This paper dealt with ideal narrowband colored-noise filters. A future study should be made into the use of an other than ideal filter. A trade off between the under and over filtering could be accomplished. This follow-on paper should also consider the use of whitening filters in place of the narrowband filter. A second area left open for further investigation is the accuracy and speed of the adaptive system to locate and follow a narrowband jammer.

Bibliography

1. Dixon, Robert C. Spread Spectrum Systems. New York: John Wiley and Sons, Inc., 1976.

2. Eaton, William G., Eugene Lawson, J. Stephen McCarthy, and August L. McGuffin. Advanced Transceiver Design Study, RADC-TR-76-175 Report 13 Feb 75 to 31 Dec 75. Utica, New York: General Electric Company, June 1976 (AD A028766).

3. Ferrari, Eugene J. and Steven M. Sussman. "The Effects of Notch Filters on the Correlation Properties of a PN Signal," Spread Spectrum Techniques, edited by Robert C. Dixon. New York: IEEE Press, 1976.

4. Giordano, Arthur. Interference Mitigation Techniques Program, PMG-616G Report 15 Jan 76 to 19 Oct 76. Needham Heights, Mass.: GTE Sylvania Inc., March 1977. (AD B017080).

5. Miller, James H. and John G. Proakis. "An Adaptive Receiver for Digital Signaling Through Channels with Intersymbol Interference," IEEE Transactions on Information Theory, 15(4):484-487 (July 1969).

6. Widrow, B., P. E. Mantey, L. J. Griffiths, and B. B. Goode. "Adaptive Antenna Systems," Proceedings of the IEEE, 55:2143-2159 (December 1967).

7. Ziemer, R. F. and W. H. Tranter. Principles of Communications: Systems, Modulation and Noise. Boston: Houghton Mifflin Co., 1976.

Appendix A

This appendix contains the calculations used to compile Table I. Table I contains the percentage value of W that B occupies, the amount of normalized degradation, the amount of power degradation associated with each value of degradation, and the percentage the spread spectrum power density function deviates from its zero frequency value. The calculations are based on a direct sequence modulated spread spectrum code with a frequency bandwidth W.

The following values are used throughout this appendix:

t_s	is equal to 2W
t_f	is equal to 1/B
sinc (t_s/t_f)	is the amount the normalized degradation value deviates from its maximum value over the interval of the correlation function.
$\text{sinc}^2(t_s B/2)$	is the factor amount the spread spectrum power density function deviates from its value at the center frequency of the filter over the filter's bandwidth.
P	is the percentage the spread spectrum power density function deviates from its value at the center frequency of the colored-noise filter over the filter's bandwidth.
d	is the amount of degradation introduced by adding the colored-noise filter into the system. It is equal to Bt_s (see Eq 20).
(1-d)	is the resulting new value of the normalized autocorrelation peak caused by the colored-noise filter.
L	is the amount of power required to overcome the loss of signal power due to the colored-noise filter and is equal to 20 log (1-d).

Table I

Effect of Closed-Noise Filter Centered on Spread Spectrum Signal

B/W (%)	d (normalized)	20log(1-d) (dB)	P (%)
0.025	.001	-0.00868	0.000
0.05	.002	-0.01738	0.000
0.10	.004	-0.03480	0.003
0.20	.008	-0.06976	0.005
0.25	.010	-0.08730	0.01
0.50	.020	-0.17548	0.04
1.00	.040	-0.35458	0.13
1.25	.050	-0.44552	0.21
1.50	.060	-0.53744	0.30
2.00	.080	-0.72424	0.53
2.50	.100	-0.91514	0.82
3.00	.120	-1.11034	1.18
3.75	.150	-1.41162	1.84
4.375	.175	-1.67092	2.49
5.00	.200	-1.93820	3.25
6.25	.250	-2.49878	5.04
8.50	.300	-3.09804	7.19
8.75	.350	-3.74174	9.68

Appendix B

This appendix contains the calculations used to compile Table II. Table II contains for each corresponding percentage value of W that B occupies, the amount of normalized degradation, the amount of power degradation, and the percentage that the spread spectrum power density function deviates from its Δf frequency value. The calculations are based on a direct sequence modulated spread spectrum code with a frequency bandwidth W.

The following values are used throughout this appendix:

t_s	equal to 2/W
t_f	equal to 1/B
Δf	equal to $\frac{W}{4}$ megahertz
f_1	equal to $\Delta f - (B)/2$
f_h	equal to $\Delta f + (B)/2$
sinc (t_s/t_f)	the amount the normalized degradation value deviates from its maximum value over the interval of the autocorrelation function
$\text{sinc}^2(t_s/f_1)$	the factor amount the spread spectrum power density function at frequency f_1 deviates from its value at frequency Δf
$\text{sinc}^2(t_s/f_h)$	the factor amount the spread spectrum power density function at frequency f_h deviates from its value at frequency f.
z	defined as $\text{sinc}^2\ (t_s \Delta f)$ and is equal to 0.405285
$\Delta 1$	equal to z minus $\text{sinc}^2\ (t_s\ f_1)$
ΔH	equal to z minus $\text{sinc}^2\ (t_s\ f_h)$
C	defined as the amplitude of the spread spectrum density function at the center of the filter, t_s z. Equal 4.05285×10^{-8}.

P the percentage the spread spectrum power density function deviates from its value at frequency Δf over the bandwidth of the filter. Equal to ΔL or ΔH, which ever is largest, divided by z and multiplied by 100.

d the amount of degradation introduced by adding the colored noise filter into the system. Equal to BC (see Eq 29).

L the amount of power required to overcome the loss of signal power due to the introduction of the colored-noise filter.

Table II

Results of Using Colored-Noise Filter Offset by 50 Percent

B/W (%)	d (normalized)	L (dB)	P (%)
0.025	0.00041	-0.00352	0.20
0.05	0.00081	-0.00704	0.40
0.10	0.00162	-0.0141	0.80
0.20	0.00324	-0.0282	1.60
0.25	0.00405	-0.03528	2.01
.50	0.00811	-0.0707	4.02
.75	0.01216	-0.10626	6.05
1.00	0.01621	-0.14196	8.08
1.25	0.02026	-0.17782	10.12

Appendix C

This appendix parallels Appendix B except the value for d is computed for the largest value of the power spectral function within the bandwidth of the filters. This corresponds to calculating d at f_1. Defining a variable, K, as

$$K = t_s \operatorname{sinc}^2(t_s f_1)$$

and using Eq (31) yields

$$d = BK$$

This gives the maximum degradation due to the filter and will serve as a maximum bound on the power lost, L, where

$$L = 20 \log (1-d)$$

From the above information, Table III was compiled.

Table III

Upper Bound on d (50 Percent Offset)

B/W (%)	d (normalized)	L (dB)	P (%)
0.025	0.00041	-0.00352	0.20
0.05	0.00081	-0.00706	0.40
0.10	0.00163	-0.01414	0.80
0.20	0.00327	-0.02844	1.60
0.25	0.00413	-0.03598	2.01
0.50	0.00827	-0.07212	4.02
0.75	0.01289	-0.11272	6.05
1.00	0.01686	-0.14772	8.08
1.25	0.02232	-0.19602	10.12

Appendix D

This appendix also parallels Appendix B. This time the lower bounds for the power lost due to the use of a colored-noise filter is being calculated. The variable Q will be used to represent the smallest magnitude of the power spectral function within the bandwidth of the filter. Q will be calculated using the frequency f_h as follows:

$$Q = t_s \operatorname{sinc}^2(t_s f_h)$$

This gives (see Eq 31),

$$d = BQ$$

where d is now the minimum degradation bound on the auto-correlation peak and thus, a minimum power loss variable.

Table IV

Lower Bound on d (50 Percent Offset)

B/W (%)	d (normalized)	L (dB)	P (%)
0.025	0.00040	-0.00352	0.20
0.05	0.00080	-0.00702	0.40
0.10	0.00161	-0.01404	0.80
0.20	0.00322	-0.02798	1.60
0.25	0.00397	-0.3455	2.01
0.50	0.00794	-0.06928	4.02
0.75	0.01144	-0.09990	6.05
1.00	0.01557	-0.13628	8.08
1.25	0.01827	-0.16014	10.12

Appendix E

This normalized energy E of a signal x(t) is defined as the energy dissipated by a voltage applied across a one-ohm resistor. An equivalent definition would be the energy E dissipated by a current x(t) passing through an one-ohm resistor. Therefore,

$$
\begin{aligned}
E &\triangleq \lim_{T\to\infty} \int_{T}^{T} i^2(t)\,dt \\
&= \lim_{T\to\infty} \int_{T}^{T} |x(t)|^2 dt \\
&= \int_{-\infty}^{-\infty} |x(t)|^2 dt
\end{aligned}
\qquad \text{(E-1)}
$$

where x(t) may be complex. Writing x(t) in terms of its Fourier transform X(f) yields

$$E = \int_{-\infty}^{\infty} x^*(t)\left[\int_{-\infty}^{\infty} X(f)\, e^{jwt} df\right] dt \qquad \text{(E-2)}$$

Reversing the order of integration to obtain

$$
\begin{aligned}
E &= \int_{-\infty}^{\infty} X(f)\left[\int_{-\infty}^{\infty} x^*(t) e^{jwt} dt\right] df \\
&= \int_{-\infty}^{\infty} X(f)\left[\int_{-\infty}^{\infty} x(t) e^{jwt} dt\right]^* df \\
&= \int_{-\infty}^{\infty} X(f) x^*(f)\, df
\end{aligned}
\qquad \text{(E-3)}
$$

The result is known as Parseval's theorem:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \qquad \text{(E-4)}$$

where

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt \quad \text{(E-5)}$$

The units for x(f) are volts-seconds. Therefore, the units for $|x(f)|^2$ are (volts-seconds) squared. On a per-ohm basis (volts-seconds) squared becomes watts-seconds-seconds. Since the inverse of seconds is hertz and watts-seconds is joules per hertz, this is the unit for energy density. Since $|X(f)|^2$ is integrated over all frequencies, the result is total energy (E_T). If x(t) is allowed to be c(t), then

$$E = \int_{-\infty}^{\infty} |c(f)|^2 df$$

$$= \int_{-\infty}^{\infty} |c(t)|^2 dt \quad \text{(E-6)}$$

For the baseband power spectrum representation, c(t), the following calculations on c(t) at radio frequency are performed,

$$E = \int_{o}^{T} [c(t)\cos(w_m t)]^2 dt$$

$$= \int_{o}^{T} c^2(t)\cos^2(w_m t)dt$$

$$= \int_{o}^{T} (1/2)c^2(t)dt + \int_{o}^{T} 1/2C^2(t)\cos(2w_m t)dt$$

$$= (1/2)\int_{o}^{T} c^2(t)dt \quad \text{(E-7)}$$

where $\cos^2 x = 1/2 + (1/2)\cos 2x$ was used. The second integral is equal to zero since the integration is over many cycles of the integrand.

The above implies that since

$$(1/2)\int_{0}^{T} |c(t)|^2 dt = \int_{0}^{T} |c(t)|^2 dt \tag{E-8}$$

where C(t) is equal to $C(t)\cos(2\pi f_m t)$

Thus,

$$\int_{0}^{T} |c(t)|^2 dt = 2\int_{0}^{T} |c(t)|^2 dt \tag{E-9}$$

From this equation, the defining relation between signals at radio frequency and signals at baseband is defined as

$$m(t) \quad = \quad 2 \quad \mathrm{Re}[m(t)e^{+j2f_m t}] \tag{E-10}$$

where m(t) is a baseband signal. Defining the total energy of the baseband signal as

$$E \; = (1/2)\int_{0}^{T} |c(t)|^2 dt$$

$$= \int_{0}^{T} |c(t)|^2 dt \tag{E-11}$$

Appendix F

This appendix compared the effects of different types of jamming interference on a direct sequence spread spectrum communication's system. Three types of jammers are considered: a wideband jammer, a narrowband jammer, and a jammer whose signal spectrum is matched to the spread spectrum signal. All three types of jammers are considered to have equal and fixed amounts of power which lie within the bandwidth of the spread spectrum signal. Figure F1 is a block diagram of the receiver.

The first type of jammer to be considered is a wideband jammer with a bandwidth equal to or greater than that of the spread spectrum signal. The jammer is considered to have its power equally spread over its bandwidth. When this interference signal goes through the first filter of the receiver, it is band limited to the spread spectrum's signal bandwidth, W. Upon going through the matched filter, it's power is spread over a bandwidth equal to two W. When it goes through the narrowband filter with a bandwidth equal to the desired signal's bandwidth, D, only a portion of the jammer's power is present at the output. Figure F2 shows this process.

In Figure F1, $J_{IN}(f)$ is set equal to $J_W/2$, the power density function of the jamming signal at baseband. $J_1(f)$ the jammer's power density function at the input to the matched filter can be calculated as follows:

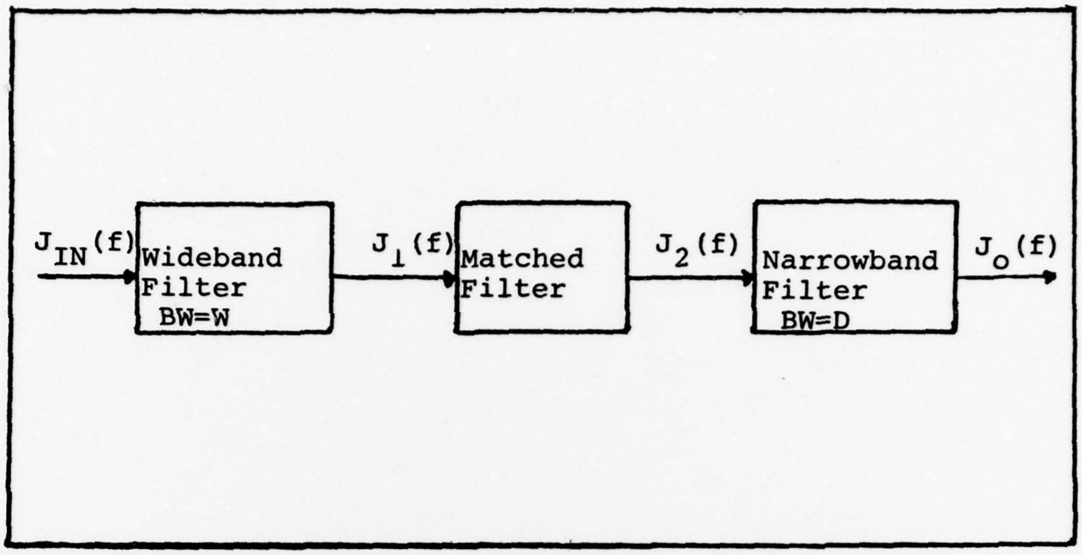


Figure F1. Block diagram of spread spectrum receiver used for noise power calculations.

$$J_1(f) = J_{IN}(f)\,|H_1|^2$$

$$= (J_W/2)\,4 \qquad -\frac{W}{2} \le f \le \frac{W}{2} \qquad \text{(F1)}$$

where

H_1 is the transfer function of the first filter at baseband

f_m is the center function of the spread spectrum signal

$J_2(f)$, the jammers power density function at the output of the matched filter, as determined by

$$J_2(f) = J_1\,|C^*(f)|^2$$

$$= 2J_W\,|C^*(f)|^2 \qquad -W \le f \le W \qquad \text{(F2)}$$

where

$C^*(f)$ is the baseband Fourier transform of the spread spectrum code

The output of the narrowband filter is given by

$$J_o(f) = J_2 \, |H_2(f)|^2$$

$$= J_2 \, 4 \qquad -D/2 \le f \le D/2 \qquad \text{(F3)}$$

To obtain the total jamming power at the output, $J_o(f)$ is integrated over all frequencies. Eq (41) is used to obtain the result

$$P_W = 8 \, J_W \int_{-\infty}^{\infty} |C^*(f)|^2 df \qquad -D/2 \le f \le D/2$$

$$= 8 \, J_W \int_{-D/2}^{D/2} S(f)\, df$$

$$= 8 \, J_W E(D/W) \qquad \text{(F4)}$$

For the narrowband case, a jammer with the same total fixed power as the wideband jammer is considered. However, the narrowband power density function $(J_N/2)$ is considered to be constant over its signal's bandwidth (B), which is within the spread spectrum signal's bandwidth (W). This narrowband jamming signal at the output of the narrowband filter of Figure F1 becomes

$$J_1(f) = (J_N/2) \, |H_1(f)|^2$$

$$= J_N 2 \qquad \text{(F5)}$$

At the output of the matched filter, the power density function becomes

$$J_2(f) = J_1(f) \, |C^*(f)|^2$$

$$= 2J_N(f) \, |C^*(f)|^2 \qquad -\frac{W+B_J}{2} \le f \le \frac{W+B_J}{2} \qquad \text{(F6)}$$

The narrowband jamming signal's power has been spread on a bandwidth equal to $W+B_J$. The jammers power density function at the output of the narrowband filter is

$$
\begin{aligned}
J_o(f) &= J_2\,|H_2(f)|^2 \\
&= 2J_N(f)\,|C^*(f)|^2|H_2(f)|^2 \\
&= 8J_N(f)S(f) \qquad -D/2 \leq f \leq D/2 \qquad \text{(F7)}
\end{aligned}
$$

Again the total output jamming power is found by integration:

$$P_N = 8J_N \int_{-D/2}^{D/2} S(f)df \qquad \text{(F8)}$$

Using Eqs (3) and (45)

$$P_N = 8J_N E(D/W) \qquad \text{(F9)}$$

Eq (F9) assumes that the narrowband filter has the same bandwidth as the narrowband jammer and allowed $J_N(f)$ to be considered a constant.

Comparing the output powers of the narrowband jamming signal to the output power of the wideband jamming signal results in the following

$$\frac{P_N}{P_W} = \frac{J_N}{J_W} \qquad \text{(F10)}$$

Since both jamming signals have equal total power and both have constant but unequal power densities over their respective frequency bands, the above equation is also equal to the inverse of their corresponding frequency bandwidths.

That is,

$$\frac{P_N}{P_W} = \frac{B_W}{B_N} \qquad \text{(F11)}$$

where

B_W is the bandwidth of the wideband jammer

B_N is the bandwidth of the narrowband jammer.

Thus,

$$P_N = \left(\frac{B_W}{B_N}\right) P_W \qquad \text{(F12)}$$

This indicates that a narrowband jammer with the same total power as a wideband jammer is more effective against a spread spectrum communications system.

The third type of jamming signal to be considered is a signal which is matched to the matched filter of Figure F1. The total energy of this jamming signal is considered to be fixed and equal to that of the wideband and narrowband jamming signals. Thus, if $j_m(t)$ is defined to be the jamming signal, then $J_M(f)$ is defined to be the Fourier transform of $j_m(t)$. Its energy spectral density is defined as $S_J(f)$, where

$$S_J(f) = |J_M(f)|^2 \qquad \text{(F13)}$$

Invoking Eq (E-10), the defining equation, to obtain the baseband representation yields

$$S_J(f) = (1/2)|J_M(f)|^2 \qquad \text{(F14)}$$

The output of the wideband filter in Figure F1 is given by

$$J_1(f) = S_J(f)|H_1(f)|^2$$

$$= 4S_J(f) \qquad \text{(F15)}$$

The output of the matched filter becomes,

$$J_2(f) = 4S_J(f)|C^*(f)|^2$$

$$= 2|J_M(f)|^2|C^*(f)|^2 \qquad \text{(F16)}$$

Since $J_M(f)$ and $C^*(f)$ are matched, then

$$|J_M(f)|^2 = G|C^*(f)|^2 \qquad \text{(F17)}$$

where

G is a proportionality constant

Thus, Eq (F16) becomes

$$J_2(f) = 2G|C^*(f)|^4 \qquad \text{(F18)}$$

The output of the narrowband filter is

$$J_o(f) = J_2(f)|H_2(f)|^4$$

$$= 8G|C^*(f)|^4 \qquad -\frac{D}{2} \le f \le \frac{D}{2} \qquad \text{(F19)}$$

The total output power is obtained by integration:

$$P_m = J_o(f)df$$

$$= 8G \int_{-D/2}^{D/2} |C^*(f)|^4 df \qquad \text{(F20)}$$

Substituting Eq (3) into the above gives

$$P_M = 8G \int_{-D/2}^{D/2} \left(\left(\frac{E}{W}\right) \mathrm{sinc}^2(t_s f) \right)^2 df$$

$$= 8G \left(\frac{E}{W}\right)^2 \int_{-D/2}^{D/2} \mathrm{sinc}^4(t_s f)\, df \tag{F21}$$

Since the narrowband filters bandwidth, D, is much smaller than the spread spectrum signal's bandwidth, W, the following approximation is made

$$D \simeq \int_{-D/2}^{D/2} \mathrm{sinc}^4(t_s f)\, df \tag{F22}$$

Thus, Eq (21) becomes

$$P_M = 8GD \left(\frac{E}{W}\right)^2 \tag{F23}$$

In order to compare the output power of the narrowband jammer to the matched jammer, it is necessary to equate G to J_N. Since the total power of both jammers are equal, the following equation can be written:

$$\left(\frac{J_N}{2}\right) B_N = \int_{\infty}^{\infty} G |C^*(f)|^2 df$$

$$= G \int_{\infty}^{\infty} |C^*(f)|^2 df \tag{F24}$$

Using Eq (E11) yields

$$\left(\frac{J_N}{2}\right) B_N = G(2E) \tag{F25}$$

Therefore,

$$G = \frac{J_N B_N}{4E} \tag{F26}$$

Substituting into Eq (23) yields

$$P_M = \frac{2D J_N B_N E}{W2} \tag{F27}$$

Comparing Eqs (F8) and (F26) gives us

$$\frac{P_N}{P_M} = 4\left(\frac{W}{B_N}\right) \tag{F28}$$

Therefore,

$$P_N = 4\left(\frac{W}{B_N}\right)P_M \tag{F29}$$

Eq (F29) indicates that a narrowband jammer is more effective against a spread spectrum communications system then a jammer with equal total power and matched to the matched filter of Figure F1. From Eqs (F12) and (F29) it is determined that a narrowband jammer is the most effective jammer against a spread spectrum communications system. The only exception is a jammer that is exactly correlated to the spread spectrum code (Ref 1:142), i.e. a duplicate of the spread spectrum direct sequence code.

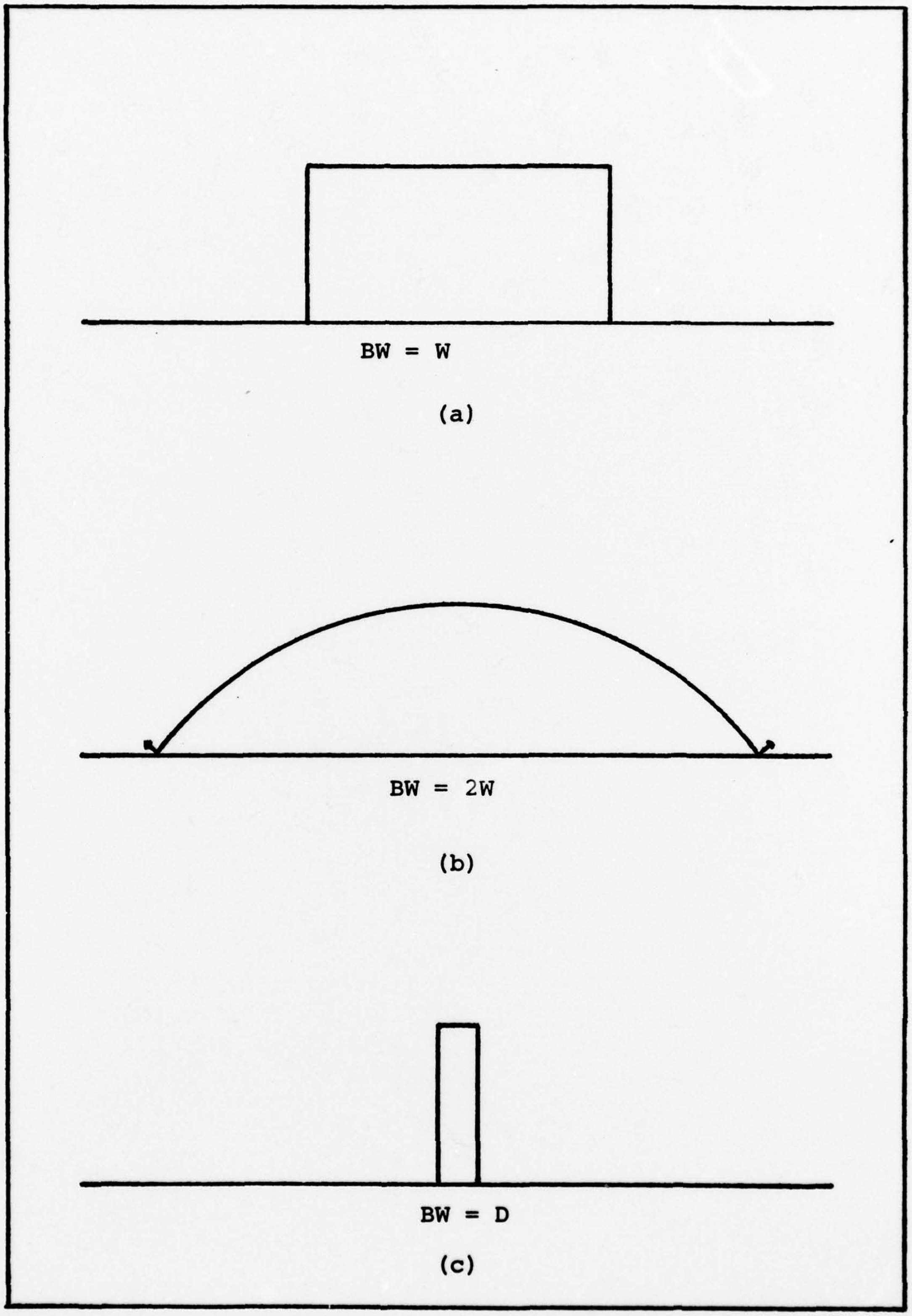


Figure F2. Plot of wideband jammer's power spectrum going through receiver of Figure F1 (a) $J_1(f)$; $J_2(f)$; and (c) $J_o(f)$.

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
AFIT/GE/EE/78D-40

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
IMPROVED ANTI-JAM PERFORMANCE IN AN ADAPTIVE SPREAD SPECTRUM COMMUNICATION SYSTEM

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
Emanuel R. Sive
Captain, USAF

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 45433

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
63727F/2277

11. CONTROLLING OFFICE NAME AND ADDRESS
Rome Air Development Center (RADC/DCID)
Griffis AFB, New York 13441

12. REPORT DATE
December 1978

13. NUMBER OF PAGES
74

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES
Approved for public release; IAW AFR 190-17

Joseph P. Hipps, Major, USAF
Director of Information 1-19-79

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
communication and radio systems
spread spectrum
adaptive systems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis investigates the anti-jam performance of an adaptive spread spectrum communication's system. The adaptive filter is assumed to be ideal in that it perfectly locates the jamming signal and adapts it's bandwidth to exactly filter out the jamming signal. The colored-noise (jammer) and band-limited white noise are considered to have flat power spectral densities within their respective bandwidths. The spread

20. spectrum technique considered is that of direct sequence modulation. Three criteria are used to evaluate the system: autocorrelation peak degradation, signal-to-noise ratio, and intersymbol interference. The results indicate that an adaptive spread spectrum communication's system is able to provide an increased jam resistant communication system.